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XVI. *A New Registering Microphotometer.* *By* Dr. W. J. H. Moll, *University of Utrecht.*

Received January 1, 1921.

(Communicated by Mr. R. S. Whipple.)

ABSTRACT.

A diminished image of a slit, on which the filament of a half-watt lamp is focussed, is projected by means of a microscope objective on the photographic plate or other object of which the absorption is to be measured. A second similar objective focusses an image of the slit, magnified up to its original size, on a second slit behind which is mounted a sensitive thermopile of the author's own design connected to an improved D'Arsonval galvanometer. By suitable gearing to an electromotor the photographic plate is given a slow motion at right angles to the beam of light, and at the same time a drum of photographic paper, on which the spot from the galvanometer is focussed, is caused to rotate at a suitable rate. A record is thus obtained of the density of the plate at different parts encountered by the beam of light. The arrangement is dead beat and so quick in response that intensity curves of close spectrum lines, Zeeman triplets, &c., are accurately recorded.

Professor Hartmann gave the name microphotometer to an instrument which he had designed to investigate the density of very small areas of a photographic plate. He compared each area with a "photographic wedge," *i.e.*, a plate of continuously decreasing density. The arrangement of his apparatus will be clear from the accompanying diagram, Fig. 1, on which the photographic plate and the wedge are indicated by P and K respectively; they receive their light by way of the mirrors S and S' from a plate of ground glass, illuminated by the small lamp, L. W is a Lummer and Brodhun double prism upon the diagonal plane of which the images of small parts of P and K are thrown by the objectives O and O'. The observer, looking through the eye-piece A, which is focussed on the centre of the diagonal plane, obtains an image of the small area of the plate P to be examined, surrounded by the image of a certain part of the wedge W. By moving the wedge, the observer may find a position such that the contrast between both images has disappeared. By reading this position and repeating the observation for other parts of the photographic plate, this latter may be investigated all over the required surface.

Hartmann's microphotometer, in its original form, was designed solely with a view to searching for areas of equal density; as such it was only a supplementary instrument in

photographic photometry. It has also been applied, however, to a problem of no less importance and of wider bearing, viz., the registering of the structure of complicated photographic pictures (*e.g.*, absorption and emission spectra).

An investigation of this kind implies, however, a long series of measurements by visual photometry. This is a tedious piece of work, and very trying to the observer's eye; notwithstanding the utmost care the precision of measurement will gradually diminish. Moreover, the fact that the observer has to compare areas of some extension renders it nearly impossible to detect delicate details, especially in objects of linear structure.

These considerations led Koch to the construction of an

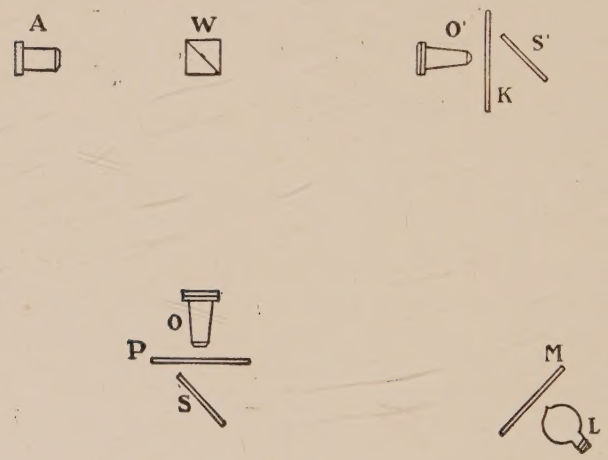


Fig. 1.

apparatus working automatically and provided with a slit; in this self-registering microphotometer Koch rejects the principle of photometry by contrast, the density being determined by directly measuring the transmitted light with the aid of a photo-electric cell. The principle of Koch's apparatus is shown in the diagram, Fig. 2. The light of a Nernst-glower N_1 is concentrated upon the photographic plate P by means of a condenser C. The objective O throws an enlarged image of the plate on the slit S_1, which cuts off all the light except that from a narrow strip of the plate P. The transmitted light falls on the cathode of a photo-electric cell F, which is connected with the negative pole of a storage-battery B. The

electric charge of the anode is allowed to flow to earth through a high resistance; the potential of the anode is measured by a string-electrometer E of the usual type. Evidently the deflections of the electrometer will more or less accurately indicate the degree of darkness of the narrow portion of the plate under observation. These deflections can be photographically recorded on a sensitive plate R by means of a second Nernst-lamp N_2, and a couple of lenses L_1 and L_2. P and R are simultaneously shifted by one and the same mechanism, P's motion being slow and horizontal, whereas R moves rather quickly in a vertical direction. In this way a

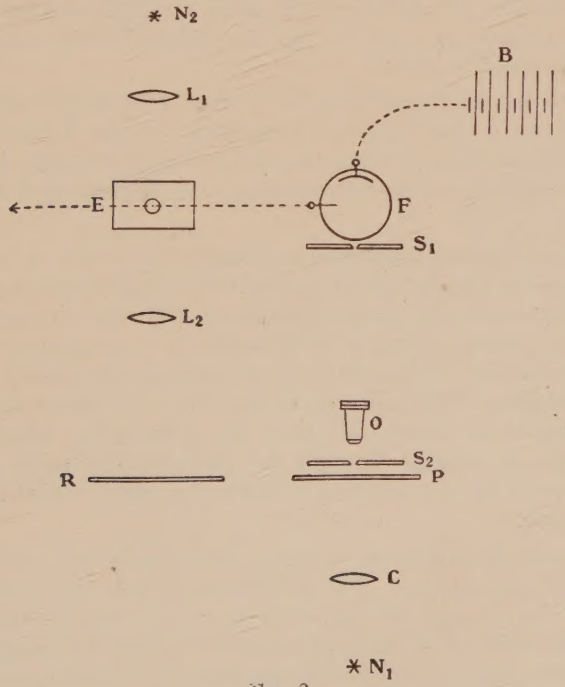


FIG. 2.

curved line is recorded on the plate R, its abscissæ indicating different places on the plate P, and its ordinates giving the degree of density at these places.

Koch himself has pointed out the difficulties encountered in the effort to obtain reliable results with this very delicate apparatus. Even if extreme care has been taken in regard to the electrostatical protection of the electrometer, the cells and

their connections, unexpected movements of the string are to be expected; the latter, moreover, is extremely sensitive to vibrations of the ground.

But the most serious objection to the method is the slowness of reaction of the photo-electric cell, which inevitably lags behind the variations of the incident light. Now abrupt changes of density seldom occur on photographic plates, and therefore, though the curve secured with a slow cell may seem to give a reasonable record of the phenomenon, this record does not represent the true distribution of density. On account of the lagging of the cell's indications a fully symmetrical spectral line, for instance, will yield an asymmetrical curve. This objection can only be met by giving the mechanism the slowest possible motion.

In all kinds of spectographic work, which have lately been carried on in the Physical Laboratory of the University of Utrecht, the need of a trustworthy microphotometer was felt. This has led me to the construction of an apparatus of my own design.

The new microphotometer has been built mainly on the same principle as Koch's, but an important improvement is effected by substituting for the photo-electric cell used in connection with an electrometer, a thermopile in combination with a galvanometer. It is now, of course, mainly the rays of greater wavelengths which are operative; but this does not make any essential difference, since the weakening which they undergo on passage through the photographic plate gives a measure of the density just as definite as when visual rays are employed.

The new arrangement, on the other hand, presents a number of great advantages. The galvanometer and the thermopile, of my own design, are instruments which exactly answer to the demands of a microphotometer. They are rapid, very sensitive, and so indifferent to external disturbances that, while in use, they do not need any special supervision.

I now proceed to the description of the new microphotometer, the principle of which will be clear from Fig. 3. In this diagram T represents a thermopile and S_1 a narrow slit. L is a lantern containing a small incandescent lamp and fitted with a condenser C, which projects the glowing spiral of the lamp upon the slit S_2. O_2 and O_1 are two microscope objectives; the former throws a greatly reduced image of the slit S_2

upon the photographic plate P, and the latter forms an enlarged image of this plate on the slit S_1. The plate P is given a slow shifting motion by a micrometer screw. The thermopile is directly connected with the galvanometer, whose mirror reflects a small spot of light on to a registering drum R. The way in which the motion of P is connected with that of R will be explained later on.

Fig. 4 gives a sketch of the whole instrument, one-third of the actual size. The base of the instrument consists of two

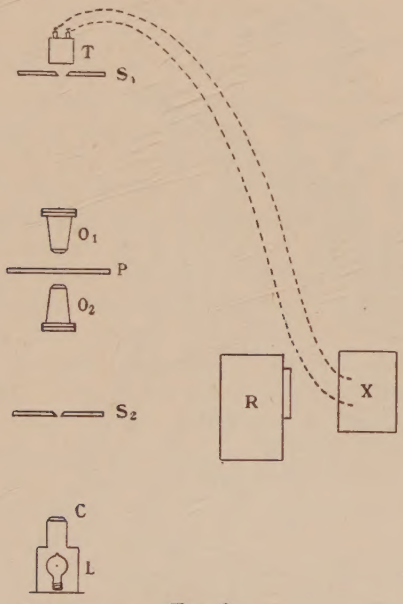


FIG. 3.

parts, mounted together on a marble slab Z: firstly, a heavy cast-bronze plate U, bearing a table D, and, secondly, an iron rail V, of triangular cross-section, along which the three bearers I, II and V may be shifted. I carries the small lantern L, II the slit S_2, and V the thermopile T. The table D carries the bearers III and IV of the objectives O_2 and O_1.

The axle A is the principal part of the motive machinery; that part of it which goes through the metal block B carries a micrometer screw with a pitch of 1 mm., which causes this block to slide between the two guiding rails F_1 and F_2. Both

the micrometer screw and its nut should be constructed with the utmost care. The plate-holder consists of two parts *M* and *N*. *N* is fixed to the block *B*, and upon it the photographic

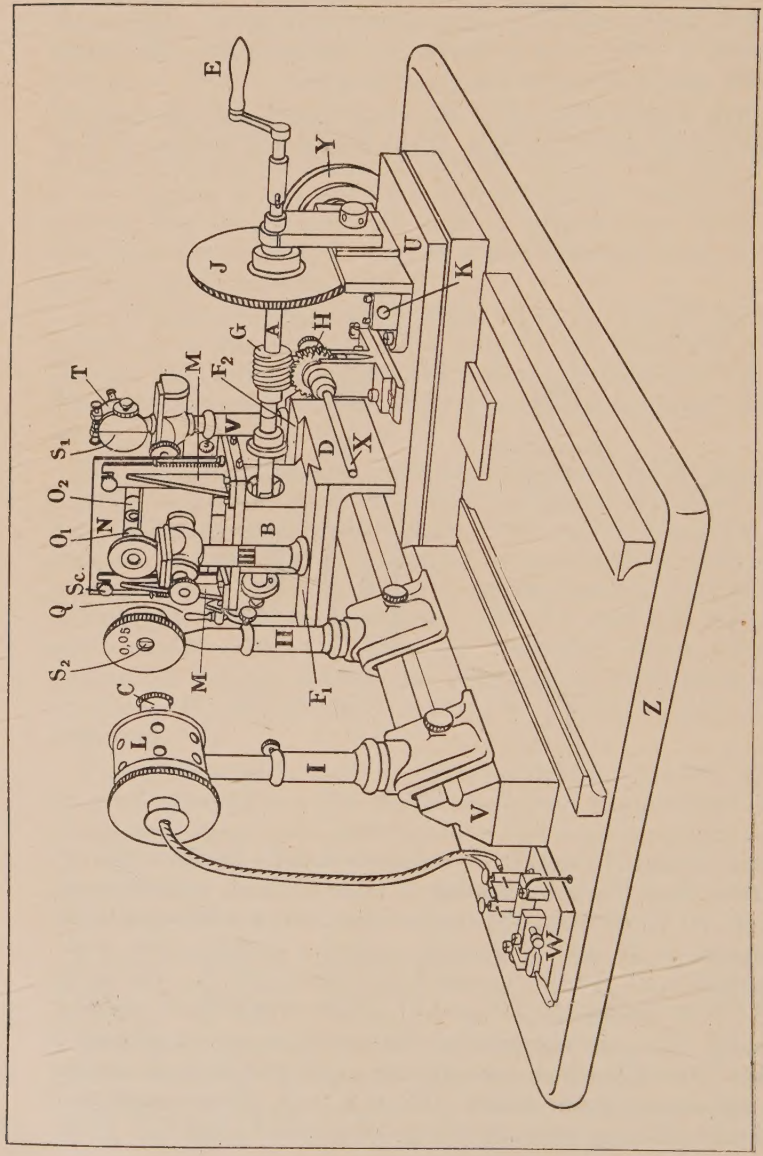


FIG. 4.

plate is rigidly clamped. The part N may be moved up and down a little by the handle Q and clamped to M by means of the screws Sc, so as to allow different parts of one and the same spectral line to be examined.

The axle A bears also a triple wormscrew G, which causes a wormwheel H to rotate at a 10-fold slower rate. The steel axle X of this wheel can be clamped to the driving mechanism of the registering drum (not shown in the sketch). This drum, whose circumference measures 40 cm., can be made to rotate either with the same speed as the wormwheel H or four times more slowly. These speeds correspond to a shift of the photographic plate of 40 and 10 mm. respectively for one revolution of the drum.

The axle A, finally, bears a second wormwheel J with 200 teeth, engaged by the wormscrew K, which by means of a triple pulley Y and a cord is driven by a moderate-speed electric motor (not shown in the sketch). By means of an adjustable resistance and a commutator the speed of rotation of the motor may be varied within wide limits and the sense of rotation reversed. Moreover, an arrangement has been devised to confine the motion of the block B between two limits; with B in either of the two extreme positions, the rotation of the axle ceases without the micrometer screw being injured. The screw K may be released from the wormwheel J and the axle turned by a handle E, in order to bring the desired part of the photographic plate between the two objectives. By doing this the registering drum, of course, is also turned, but it can be replaced in its original position by a special device.

The optical arrangement of the microphotometer has already, in its principal lines, been explained in Fig. 3. For further particulars we may refer to Fig. 4. The thermopile T, protected by a glass window of 6 mm. diameter, is mounted in a heavy brass cylinder. The jaws of the slit S_1 are formed by two half-circles of white celluloid, providing a surface on which careful focussing is possible. The width of the slit can be adjusted by a screw with divided head. The lantern L with condenser C holds a small incandescent lamp with special fitting, which secures a steady position of the lamp. A key W of special design ensures a quite reliable electric contact. The two objectives are adjustable in the direction of the optical axis; the thermopile and slit S_1 may be shifted sideways.

The microphotometer is adjusted as follows: Remove the slit S_2 and bring the objective O_2 close to the photographic plate; a relatively large area of this plate is then illuminated. Now produce a sharply defined image of the plate on the jaws of the slit S_1; this leads to the correct position of the objective O_1. If a line-spectrum is to be examined, care must be taken, by turning the slit S_1 a little, that it runs parallel to the projected lines. Replace the slit S_2, and let a sharp image of the spiral of the lamp be formed upon it. Next move the objective O_2, and turn the slit S_2 so as to obtain a sharp image of it on the slit S_1, running parallel to the latter. Finally, shift S_1 into coincidence with the image of S_2, and open the slit to the required width.

In contrast to S_1, the slit S_2 has fixed jaws; it can, however, be replaced by others, so that the observer may have at disposal four different widths, *viz.*, 0·05, 0·1, 0·25 or 0·4 mm.

One might suppose that the slit S_2 and even the condenser C might as well be omitted, so as to project an image of the spiral directly on the photographic plate. It is true that this simpler arrangement would yield even greater deflections of the galvanometer. But omitting the slit S_2 would entail serious diffuse scattering of the light in the sensitive film, so as to injuriously affect the delicate details of the phenomenon under consideration. In Koch's microphotometer this danger is not met by use of the slit, which he places *behind* the photographic plate (S_2 in Fig. 2); this slit only improves the sharpness of definition by partly shutting off the "false light." In my instrument the slit S_2 not only cuts off the false light totally, but also reduces the unavoidable scattering by the photographic plate to a minimum.

As to the choice of a suitable place for the instrument, no special conditions are necessary. It is important that the galvanometer should be free from mechanical vibration; but this can be secured merely by mounting it on an ordinary bracket attached to a solid wall. The registering apparatus need not be placed in the dark, if care is taken that no direct light can fall on the cylindrical lens in front of the drum.

In conclusion I should like to show some of the results obtained with, the new microphotometer, which may give an idea of its efficiency.

Fig. 5 is the reproduction of a photogram yielded by a Zeeman sextet; the photographic plate was kindly provided by Professor P. Zeeman. The sextet, obtained with a spark

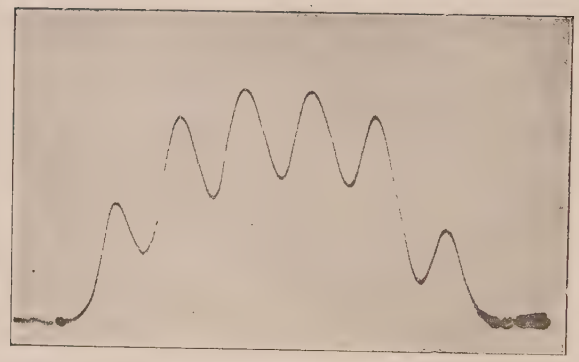


FIG. 5.—PHOTOGRAM OF ZEEMAN SEXTET.

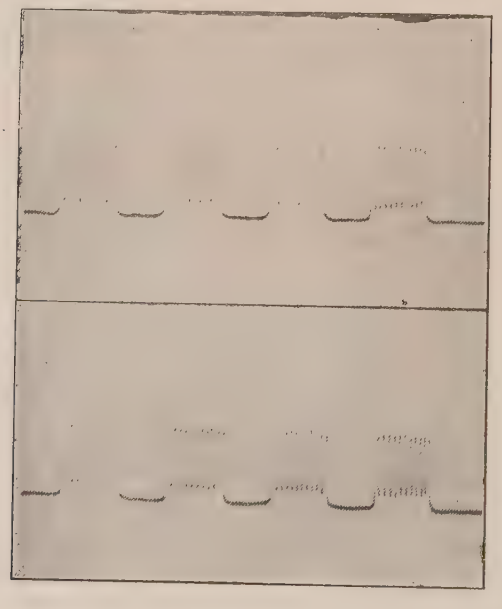


FIG. 6.—PHOTOGRAM OF ABBEY TEST-PLATE.

To face p 215.]

between Na-electrodes, had a total area of about 1 mm.^{2} Fig. 6 represents the photogram yielded by an "Abbe test-plate." This is a glass plate with a layer of silver in which 4 series of 11 lines each have been engraved; each line has a width of about 0·02 mm., and the space between two adjacent lines is also about 0·02 mm; the lines have been irregularly engraved on purpose. The photogram shows very clearly the differences between the successive lines. The object has been recorded twice in succession; the two curves are sensibly identical.

DISCUSSION.

Mr. R. S. WHIPPLE thought that the dead-beatness and quickness of the apparatus were unequalled. There was a beautiful feature of the records to which Dr. Moll had not referred; they showed the actual texture of the photographic plate as ripples on the base line. He believed the author was engaged in another research which would throw light on some important problems in connection with the blackening of photographic plates. It would be difficult for most of those present to imagine that the record of the Zeeman effect shown had been taken inside half a minute. It was a triumph of experimental skill.

Prof. RANKINE said he had had the privilege of being about during the last few days when Dr. Moll and Prof. Ornstein had been erecting the apparatus. He had suggested that the instrument might be used to settle the question of the existence of the canals on Mars. In this connection Dr. Moll had told him that the eye saw many things which did not exist and missed many things which did. An example which might be of interest to radiologists present was the case of an X-ray photo of a square hole in an opaque screen. It was frequently asserted that the intensity was greater at the edges of such a square than at the centre. The thermopile showed that this was wholly an illusion.

Mr. GUILD said he could only express the greatest admiration for the instrument they saw before them. He was particularly interested in the galvanometer. They had recently been installing radiometric apparatus in the Optics Department of the National Physical Laboratory, and were using a Paschen galvanometer. They were troubled so much with the zero changes due to stray magnetic fields from the adjacent electric railways and from sources within the laboratory that the advantages due to the sensitivity of the galvanometer were largely nullified. He looked forward with very great interest to the advent of Dr. Moll's thermopiles and galvanometers on the British market.

Dr. J. S. ANDERSON said he had had the pleasure of hearing Koch describing his microphotometer. He had thought the chief objection to Koch's instrument was the variability of the photo electric cell. The modern forms were much less variable and probably the objection did not now hold to the same degree. Dr. Moll was, however, to be congratulated on so successfully employing the much more easily manipulated thermopile for the purpose.

Mr. C. R. DARLING thought the instrument would be readily adapted to form a recording radiation pyrometer.

Mr. F. E. SMITH said he would like to know how Dr. Moll had made such an excellent thermopile. Through the kindness of Mr. Whipple he had had the pleasure of working with one of them, and there was no question of its excellence, but he had not been able to see how it was made.

Dr. Moll, in reply, said the secret of his thermopiles consisted in making the junctions which received the radiation of very low heat capacity. With most thermopiles the bulk of the heat capacity was due to the solder at the junctions. His method of avoiding this was as follows : He took two thick plates of the metals to be used, constantin and manganin, and soldered them end to end, scraping off the excess of solder. The metals already joined in this way were then rolled out to the desired thinness, and cut into strips. In this way strips only 2μ thick were obtainable with no excess of solder at the junctions. The other junctions were made of large heat capacity to give constancy of temperature of the "cold" junctions.

As regards the galvanometer, the principal modification which he had introduced was to have the coil held by two strips of bronze in tension instead of suspending it by the upper strip in the usual way. This gave greatly improved constancy of zero.

XVII. *A Balance Method of Using the Quadrant Electrometer for the Measurement of Power.* *By* HUBERT PARRY.

RECEIVED JANUARY 1, 1921.

(COMMUNICATED BY R. S. WHIPPLE, ESQ.)

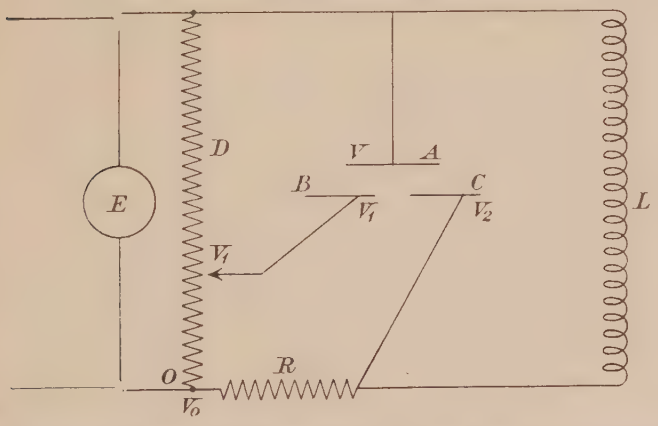
ABSTRACT.

A new method of using the quadrant electrometer for the measurement of electrical power is described. It involves the use of a potential divider across the supply circuit, and a standard non-inductive resistance in series with the load. An ammeter and a voltmeter are employed to measure the supply voltage and the load current.

An example of its application to determine the power-factor of a condenser is quoted.

INTRODUCTION.

ANYONE who has used a quadrant electrometer as a deflectional wattmeter will know the difficulties in getting this instrument to obey the " Heterostatic Law " over fairly large angular deflections, and in keeping it so from day to day ; it requires very great accuracy in workmanship so that the instrument shall be free from electrical control with the vane or needle in any position between the quadrants. It was to



get over these difficulties that I conceived the method described in this Paper.

Method.—The electrometer is used as a balance indicator, as shown in the figure, where A represents the needle, B and C the quadrants, R a noninductive resistance in series with the load L, D a calibrated potential divider coupled between the potential point O on the supply side of R and the far side

of the load, to which point the needle A is coupled. E is a voltmeter.

It is, of course, necessary that the instrument shall be free from electrical control: that is, that the zero shall not change at any voltage over the working range, when the voltage is applied to the needle and the quadrants shorted; this condition is easily got by slightly tilting the instrument.

Now, if a voltage be applied to the needle A, and we have a voltage on quadrant C proportional to the current, we can apply with the potential divider D a voltage in phase with the needle voltage to quadrant B, so that the average torque in the instrument is zero. As the instrument is used it will obey the "Heterostatic Law" and, should the power-factor of the circuit be unity, the voltages on B and C will be equal, hence the product of the voltages on the needle A and quadrant B will be proportional to the watts expended in R and L. If the power-factor is not unity, then the average voltage on B will be less than on C, but a voltage can be applied to B such that the average torque is zero.

Theory.—

Let v be the instantaneous voltage on the needle A,
,, v_1 ,, ,, ,, ,, quadrant B,
,, v_2 ,, ,, ,, ,, quadrant C,
,, v_0 ,, zero potential at point 0,
and ,, R ,, resistance of R; where v, v_1, v_2 are periodic functions of time.

The theoretical law of the electrometer is given as

$$\text{Torque}=K\{v-\tfrac{1}{2}(v_1+v_2)\}(v_1-v_2)$$

where K is the instrument constant. As the needle is always in the same position this law will hold good.

Hence the instantaneous torque (t)

$$=K\{v-\tfrac{1}{2}(v_1+v_2)\}(v_1-v_2),$$
$$=K[vv_1-vv_2+\tfrac{1}{2}(v_2^2-v_1^2)].$$

Then by integrating over a complete period and taking the average we have the average torque (T)

$$=K[VV_1-VV_2\cos\varphi+\tfrac{1}{2}(V_2^2-V_1^2)]=0,$$

where V, V_1, V_2 are square root of the mean square values and $\cos\varphi$ the power-factor of the circuit.

If we divide the above expression by R and transpose terms, we have

$$\frac{VV_2}{R}\cos\varphi = VI\cos\varphi = \frac{VV_1}{R} + \frac{1}{2R}(V_2^2 - V_1^2) =$$

power expended in the resistance R and load L, where I is the R.M.S. current in the circuit.

As an example, and to show the fair agreement with results got with a standard Duddell wattmeter, a 1 mfd. condenser was tested at 90 volts, 40 cycles, for losses at air temperature.

The Duddell instrument gave :—

Watts, 0·0039 ; power-factor, 0·0012, at 92 volts, 40 cycles.

The method described gave :—

(1) Watts, 0·0040 ; power-factor, 0·0019 at 90 volts, 40 ∿, 11°C.

(2) Watts, 0·0030 ; power-factor, 0·0015 at 90 volts, 40 ∿, 11°C.

Below are given the detailed figures :—

V.	Potential divider ratio.	V_1	V_2	R	Freq.	Milli-amps.	Temp.
(1) 90·1	$36{\cdot}7\times10^{-5}$	0·033	$2{\cdot}26_6$	100	40·25	$22{\cdot}6_6$	11°C.
(2) 90·75	$35{\cdot}3\times10^{-5}$	0·032	$2{\cdot}28_2$	100	40·25	$22{\cdot}8_2$	11°C.

The total resistance of the potential divider is 100,000 ohms, and the division can be altered by unit steps, keeping the total resistance constant. The first decimal place is by estimation.

This is quite good when one considers the low power-factor, and that the instrumental losses are of the same order as those to be measured. In the above test the voltage V was measured by a reflecting electrostatic voltmeter where the sensitivity is such that 8 cm. of scale represent 1 volt. V_1 is calculated from potential divider data and V; V_2 from the current and resistance R, or measured with an A.C. potentiometer.

Conclusion.

It will be seen that the voltmeter is the only instrument needing a scale ; these instruments can be made with strong control and will keep their accuracy from day to day. The potential divider and series resistance need no special comment, except to say that the potential division must be true as regards phase as well as magnitude. The advantages are

that no wattmeter scale is needed, and therefore no calibration, and the electrometer does not need to be so accurately made as for the deflectional method. The voltmeter and potential divider would still be needed for calibration if the electrometer was used as a deflectional instrument.

The disadvantages are, slowness of working, and the need to have a very steady supply for accurate work.

I have used the method for over a year with quite good results, and as far as I am aware it is new. I have published it in the hope that it may be of interest and use to others.

DISCUSSION.

Mr. C. C. Paterson said he had worked for 10 to 15 years with electrometers for power measurements. He would like to see the method tried out more fully under varying conditions. With small power-factors one always had the result as the difference of two large terms when using the electrometer.

Another point was that the two voltages V_1 and V_2 were not in phase. Of course, V_2 was small but it would surprise him to find that it was legitimate to add them as the author had done.

Dr. E. H. Rayner said they had not yet tried the method at the National Physical Laboratory, but were certainly going to. He wondered if the difficulty mentioned in connection with small power-factors could not be overcome by connecting the needle to the mid-point of a large dividing resistance. There was always a danger, however, with small power-factors that the dividing resistance was not true as regards phase on account of the distribution of capacity.

Dr. A. Russell congratulated the author. The method was novel to him. The old method was quite good for high voltages, but for low voltages he thought the author's method would be superior.

Dr. D. Owen said: The author's Paper, of modest dimensions, contains an idea which promises to add decidedly to the value of the quadrant electrometer as used for the measurement of electrical power. The method, being a null one, possesses several advantages. In the first place, it makes it possible to utilise to the full the possession of an electrometer of the highest sensibility attainable in virtue of mechanical design. Further, as the constant of the electrometer disappears from the expression for the power, the usual calibration experiment with non-inductive load becomes unnecessary.

In the form proposed, however, the method has one decided disadvantage, namely, that at low power-factors the power required was obtained as the difference of two separately determined quantities, each of which might be of much greater magnitude. This is so, indeed, in the very test chosen by the author for illustration, and the effect is evident in the considerable discrepancy between two successive tests on the same condenser.

Dr. Rayner has expressed the possibility of this source of error being overcome by connecting the needle to the mid-point of the non-inductive resistance across the mains. This had occurred to me, and on trying it I found it to be the case. The adoption of this modification would not only eliminate the source of error occurring at low power-factors, but would simplify the expression for the power in the general case. Denoting by V/n the voltage drawn from the potential-divider (denoted by V_1 by the author) the expression for the power supplied to the load becomes

$$W = \frac{V^2}{R} \cdot \frac{n-1}{n^2}$$

All that is required in a test now is to adjust the tapping-point on the potential-divider until the electrometer needle is at zero, and at that instant to read off V on the voltmeter. n may be observed at leisure, and the only remaining factor in the calculation of the power is the standard non-inductive resistance R.

Mr. PARRY communicated the following reply :—

The theory of the method is based on the conception of the electrometer acting as two instruments in opposition—the one having elements A and B working at unity power factor with potentials V, V_1 and V_0; the other with elements A and C working at the power-factor of the load and potentials V, V_2 and V_0. If Mr. Patterson will look at it from this viewpoint I think he will find the latter part of his remarks answered.

As pointed out by Dr. Owen and Dr. Rayner, the difficulties at low power factor are eliminated by making the needle potential $\frac{V}{2}$; it had not occurred to me to do this, though I knew of this property in the deflectional methods.

Dr. Owen's formula can be expressed in a slightly more convenient form by taking $V_1 = KV$; then

$$W = \frac{V^2}{R}(K - K^2).$$

K can generally be read directly on a potential divider.

The method has checked out very well with results got by dynamometer wattmeters working over a power factor range of 1 to 0·3. I shall be very pleased to hear the N.P.L. opinion after they have tried the method.

In conclusion, I should like to thank Dr. Owen for the large trouble he has taken over my small Paper.

XVIII. *Application of the Ionisation Spectrometer to the Determination of the Structure of Minute Crystals.* *By Prof. Sir* W. H. Bragg, *F.R.S.*

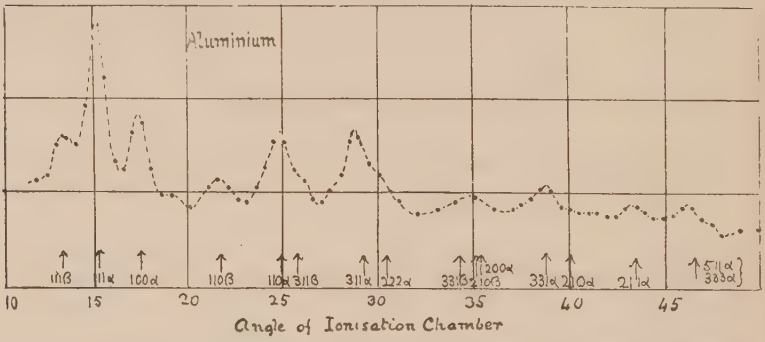
Received March 31, 1921.

ABSTRACT.

It is shown that crystals in the form of powder can be examined by the ionisation method. The powder is pasted on a flat surface and placed on the spectrometer table in the position ordinarily occupied by the face of a single crystal. A bulb current of 1 milliampere is sufficient to give satisfactory records.

In the first determinations of crystal structure by the help of X-rays large single crystals were employed; and the reflections were observed by means of the ionisation effects of the rays. It is not convenient, however, to handle in this way crystals weighing less than a few milligrams. The great majority of substances do not yield crystals even as large as that.

Debye and Hull have independently developed a method of dealing with crystals in a state of minute subdivision. A

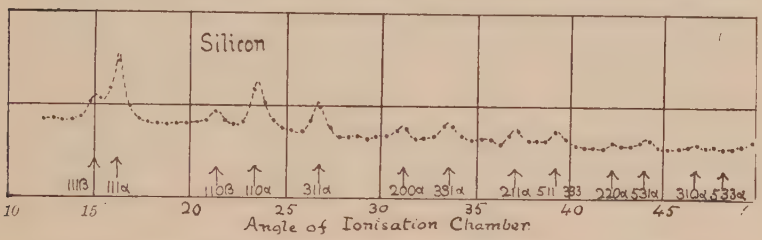


monochromatic beam of X-rays is passed through a small quantity of the powder; some of the small crystals are oriented so as to yield each and every one of the reflections proper to the particular wave length and the particular crystal. The results are recorded photographically; and it is usual to give long exposures with heavy discharges in the X-ray bulb. This is necessary because only a small fraction of the crystalline matter is at any one moment so oriented as to reflect.

A few instances given below show that crystals in the form of powder can be examined satisfactorily by the ionisation method. The powder is pasted on to a flat surface and

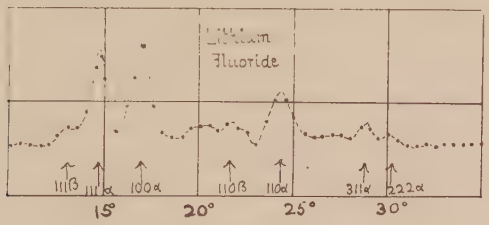
placed on the table of the spectrometer in the position ordinarily occupied by the face of a single crystal. The slit of the spectrometer may be wide enough to take in reflections over half a degree. The intensity in different parts of the spectrum can be examined in detail. Each reading occupies a few seconds and the main part of the spectrum can be run through in half an hour. A bulb current of one milliampere is sufficient.

The method has some advantages of its own. It measures



relative intensities without further calculation, and avoids the difficulty of interpreting the blackening of a photographic plate.

The aluminium spectrum shown in the figure displays the reflections characteristic of a face-centred cubic lattice, and of the simple piling of atoms into a close packed structure. The corresponding photographic spectrum is given by Hull in the "Physical Review," X., 1917. The rays used in finding



spectra here given were those of the K series of rhodium. The silicon spectrum is characteristic of the diamond lattice; its most obvious difference from that of aluminium is the absence of the 100 line. The spectrum of lithium fluoride is notable for the peculiarity in the relative magnitude of 100α and 111α. If the lithium atom had been quite negligible compared to the fluorine atom the spectrum would have been

similar to that of aluminium, with its large 111 reflection. But the lithium atom is not negligible, though small; and so the 111 spectrum is considerably reduced. In rock-salt it is known that the 111 is quite small: in that case the metal atom is larger in comparison to the halogen atom.

DISCUSSION.

Mr. J. Guild said that while listening to this Paper, with the preceding Paper by Dr. Moll on improved thermo-electric measurements still in his mind, he had been wondering whether the modern improvements in the methods of X-ray production and of radiometry had possibly brought us within reach of direct measurement of the energy of X-radiation. Using a sensitive thermopile with lead foil receiving plates, and a heavy discharge from a Coolidge tube, there might be sufficient energy stopped by the foil, although most of it would pass through, to give an indication on the galvanometer.

Prof. Ornstein thought that insufficient energy would be stopped by the thermopile to produce any effect unless the lead was so thick that all the sensitivity of the pile would be destroyed.

XIX. *The Effect of Viscosity on Orifice Flows.* By W. N. Bond, *B.Sc.* (*Engineering*), *M.Sc.*(*Lond.*), *A.R.C.S.*, *A.Inst.P.*

Received November 16, 1921.

ABSTRACT.

Determinations were made of the coefficient of discharge through an orifice 0·1469 cm. in diameter of solutions of glycerine and water, varying in kinematic viscosity from 0·01 to 7. The results are plotted in a manner which combines both purely viscous and purely turbulent flows in one graph. It is shown that the effect of slight viscosity is to increase the coefficient of discharge.

A large number of experiments have been carried out by different workers on the flow through orifices when the effect of viscosity can be neglected. When viscosity has to be taken into account, the value of the coefficient of discharge C, given by $C=Q/(A\sqrt{2gh})$, should be the same for all cases having the same value of $vl\rho/\mu$,

where Q=observed volume passing per second.
h=the difference of "head" between the two sides of the orifice.
A=the area of the orifice.
$v=Q/A$=the average velocity at the orifice.
l=the diameter of the orifice.
ρ=the density of the liquid.
μ=the viscosity of the liquid.

The kinematic viscosity μ/ρ may be denoted by ν.

The following experiments were carried out for J. L. Hodgson, Esq., B.Sc., A.M.I.C.E., of Messrs. Geo. Kent, Ltd.; the object of the investigation being to obtain the values of C for lower values of $vl\rho/\mu$ than were previously available, in the case of a circular sharp-edged orifice in a thin plate, having liquid on both sides of it.

It was known from previous experiments that as viscosity becomes more important the value of C is at first increased; and it is necessary that for yet more viscous liquids C should decrease to zero. Since the completion of these experiments the author has found that G. F. Davidson* had previously attempted to obtain similar information in the case of a round edged orifice. For his experiments a thick engine oil was used, the viscosity being varied at will by changing the temperature.

* Proc. Roy. Soc., A., Vol. LXXXIX., 1913-14, pp. 91-99

The values obtained for the coefficient of discharge were not constant for any one value of $vl\rho/\mu$, and he traced the cause of the discrepancy to the fact that the oil he was using had not a definite value of ν independent of the rate of shear. The value of ν was found to decrease for an increase in the rate of shear. Similarly an increase in the value of ν when the rate of shear is made small has been found by Kurt Molin,* using Searle's viscosity apparatus.

In the present experiments, at Mr. J. L. Hodgson's suggestion, mixtures of glycerine and water were used, the value of ν being thus varied from about 0·01 to 7.

The viscosity of each mixture used was determined by the capillary tube method, the capillary tube being placed in a

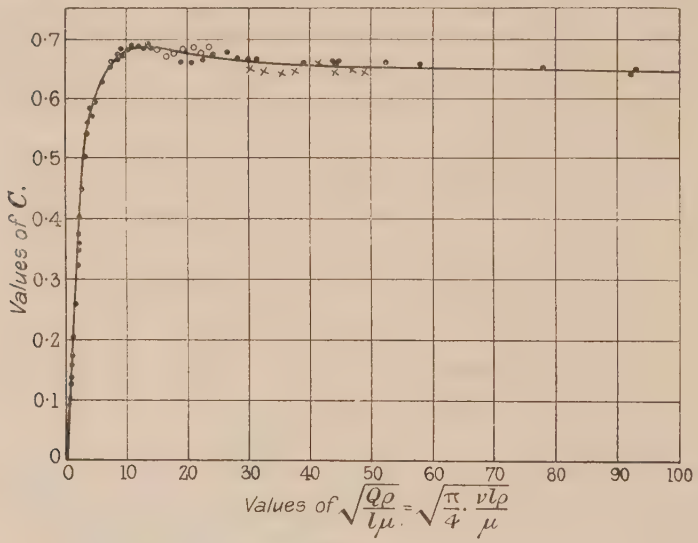


FIG. 1.

water bath. The values obtained were consistent and agreed in the cases of water and pure glycerine with the values usually accepted.

The density of each mixture was determined by means of a hydrometer.

The orifice was made by clamping a piece of thin hard-rolled sheet German silver between two gunmetal plates and drilling through the three together. After unclamping, the

* Kurt Molin, Proc. Camb. Phil. Soc., Vol. XX., Pt. I., pp. 23-34, 1920.

orifice plate was polished on both sides to ensure the edge being right-angled. This plate was then soldered to a small gunmetal ring, which in its turn was fastened to one end of a wide glass tube with plasticine. The wide glass tube, which was about 76 cm. long by 2·85 cm. diameter, was clamped vertically with the orifice at the lower end. This end was arranged to dip into a large beaker containing some of the liquid being used. A scale divided in millimetres was pasted along the length of the tube. The area of cross-section of the tube at different points was obtained by closing the lower end, pouring in successive known volumes of water and noting the corresponding rise in level.

The diameter of the orifice was measured by means of a microscope fitted with an eyepiece scale micrometer. Check measurements were obtained by other means. The mean of the determinations was 0·1469 cm. diameter. The thickness of the orifice plate was obtained by means of a micrometer screw gauge, which gave a value of approximately 0·0075 cm.

In the flow experiments the vertical tube was filled with liquid, the orifice being meanwhile closed by one finger. An air-tight cover was placed on the top of the tube to prevent flow taking place till desired. The beaker was put in position and any air bubbles removed from the orifice plate. Time was allowed to let any bubbles in the liquid rise to the surface. On removing the top cover the time was noted when the falling liquid surface in the tube passed suitable divisions on the scale. The level in the beaker was read when desired, and a correction for its rise applied to the value of the "head." The temperature of the liquid was observed and a viscosity test carried out at the same temperature.

From the determinations of the time of passing successive divisions on the tube scale, the rate of flow could be deduced, since the area of cross-section of the tube had been obtained by the preliminary experiment. This very convenient method of experimenting was suggested by Mr. F. Gray.

For the timing in the case of the less viscous mixtures a chronoscope reading to 1 50 sec. was used; and it was found that successive readings could be taken satisfactorily at intervals of as little as six seconds, the observing and writing being carried out by two observers.

In all the more viscous mixtures, great care had to be taken to remove air bubbles. Also in the case of the most viscous liquids there was an appreciable quantity of liquid left clinging

to the inside of the glass tube. The error this would cause was made small by the use of the small size of orifice adopted.

The space between the orifice and the bottom of the beaker was varied in the case when $\nu=0{\cdot}35$ from 2·2 cm. to 4 cm., and no measurable change in C occurred. For most of the experiments the length of this free downstream was 2·2 cm. which is about 15 times the orifice diameter.

At the end of this set of experiments the orifice plate was fitted into a pipe line and the value of C obtained for larger water flows.

In plotting the results, instead of plotting the values of C

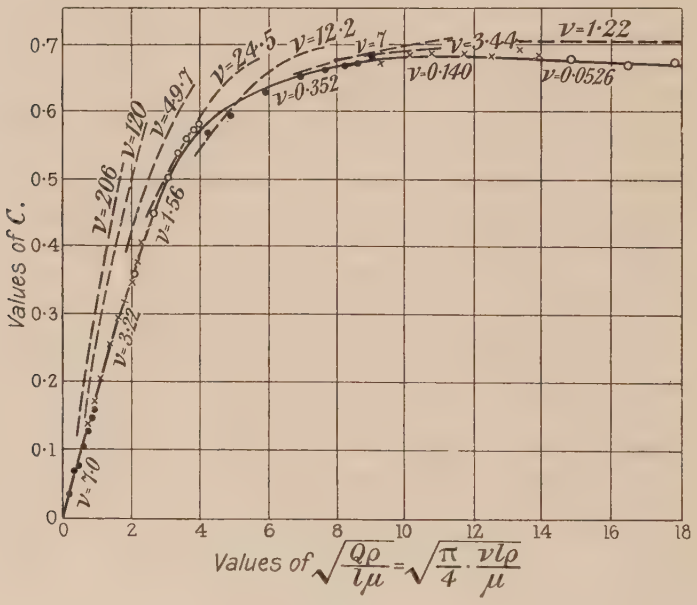


Fig. 2.

against those of $vl\rho/\mu$ or its equivalent $4Q\rho/\pi l\mu$, it was found better to plot against $\sqrt{Q\rho/l\mu}$ in order to obtain a more convenient representation of both viscous and turbulent portions of the curve on a single graph. This method of plotting has also the advantage that it causes both the purely viscous and the purely turbulent portions of the graph to become straight lines.

Fig. 1 shows the curve obtained by plotting the results on this basis. In Fig. 2 the portion corresponding to viscous

flow is plotted on a larger scale. The curves given by C. F. Davidson* in Fig. 2 of his Paper have been reduced to the same basis and are indicated by the broken curves in Fig. 2. The values of the kinematic viscosity $\nu = \mu \rho$ are given by each curve. The figures above refer to Davidson's experiments; those below to the present results. The points obtained with different values of ν are distinguished by plotting successive sets with dots, crosses, circles, and so on. In Fig. 2 the viscous flow determinations (shown in detail in Fig. 2) are for simplicity indicated entirely by dots.

It will be noted that the extreme values of ν used are in ratio of about 1 to 700 (cf. Davidson's 1 to 1,000). Moreover, the results obtained with the more viscous mixtures lie extremely closely along a straight line through the origin. This indicates that true viscous flow is obtained; and further that under the conditions of these more viscous experiments the viscosity of the mixtures is to a close degree independent of the rate of shear. However, when less viscous mixtures were used, there are indications of successive experiments with different amounts of dilution showing slight disagreement; and it is probable that for the less viscous mixtures (and consequently greater rates of flow) the value of ν is no longer independent of the rate of shear. Thus though the left-hand end of the curve is probably determined with fair accuracy, the shape of the apex may be slightly in error, due to variations in ν similar to those observed by Davidson. Any error due to this cause is probably confined to the portion of the curve near the apex, since the linear law agreement obtained for purely viscous flows renders error there improbable; and for purely turbulent flows any variation in ν with rate of shear can have no effect on the resultant curve.

The fact that viscosity, when its effect is small, produces an increase in the coefficient of discharge (perhaps due to its causing an increase in the coefficient of contraction) is in agreement with previous experiments carried out at Messrs. Geo. Kent's, and with the results obtained by Davidson in the case of water. The results obtained by Osborne Reynolds† and subsequent experimenters for the pressure gradient in the case of flow in pipes, show a similar increase in the discharge caused by slight viscosity. Another set of experiments showing a similar result is the determination at the National Physical

* Proc. Roy. Soc., A., Vol. LXXXIX., 1913-14, p. 94.
† Osborne Reynolds, Phil. Trans. Roy. Soc., 1883.

Laboratory of the force exerted on a cable by a transverse air current.*

In conclusion the author would like to thank Messrs. Geo. Kent, Ltd., for the facilities that have made these experiments possible; Mr. J. L. Hodgson, at whose instance the experiments were made, for his kindness in suggesting such an interesting problem for investigation; and Mr. F. Gray, conjointly with whom the tests were made, for his kindly co-operation throughout the progress of the work.

DISCUSSION.

Mr. A. S. E. Ackermann said he had made experiments of a similar character in which the substance tested was clay. The clay was packed in a box with an orifice of from 3 to 20 millimeters diameter in the bottom. The space above the clay contained air, and the pressure was raised by means of a pump.

The speaker then drew curves on the board to show his main results. With any particular orifice a certain initial pressure is required before any clay is forced out. As the pressure increases the rate of the flow becomes more rapid, and finally becomes very rapid indeed. The critical pressures and the orifice diameters were connected by the law $pd^{0\cdot833}=5408$ for a clay with 29 per cent. water. Similar results are obtained in forcing a disc through clay. The speaker mentioned that these and other results were given in a Paper by him to the Society of Engineers, which was at present in the Press.

Mr. Bond said the phenomena described by Mr. Ackermann were probably due to plasticity, rather than viscosity.

* Report of the Advisory Committee on Aeronautics, 1912-14.

XX. *Viscosity of Water at Low Rates of Shear.* By Albert Griffiths, *D.Sc.*, *and* Constance H. Griffiths, *M.Sc.*

Received January 27, 1921.

ABSTRACT.

This Paper describes the determination of the coefficient of viscosity of water by a method in which water is forced along glass capillary tubes of about 1·5 to 2·0 mm. bore at rates of flow varying from 1 litre in two years to 1 litre in 24 years. The liquid fills a closed tubular circuit which for purposes of description may be said to be rectangular in shape. Two of the tubes of the rectangle are horizontal, and two vertical. The circulation is caused by a difference of density between the contents of the two vertical tubes. The difference is obtained by having a weak solution of uranine in one vertical tube, and pure water in the other.

In Apparatus I. the two vertical tubes are at a distance apart, and an allowance has to be made for the difference of temperature. In Apparatus II. a modification of the rectangular circuit is employed, the vertical tubes are close together, and no such correction is necessary.

Applying the temperature correction to the results recorded in this Paper, the value of the co-efficient of viscosity at 18°C. is 0·010624 ± 0·000229 (assuming observations of equal weight) and 0·010569 gm./cm. sec. from Hosking's formula. Thus there is no experimental evidence that at the extremely low rates of shear (0·0017 to 0·0233 radians/sec.) the viscosity of air-free water in glass capillary tubes differs from its value at normal rates of shear (5,000 to 10,000 radians/sec.).

§1. Introduction. §2. Apparatus I.
§3. Apparatus II. §4. Results.

§1. Introduction.

This Paper describes the determination of the coefficient of viscosity of water by a method in which water is forced along glass capillary tubes of about 1·5 to 2 mm. bore at rates of flow varying from 1 litre in two years to 1 litre in 24 years.

The liquid fills a closed tubular circuit which for purposes of description may be said to be rectangular in shape.

Two of the tubes of the rectangle are horizontal and two vertical. The circulation is caused by a difference of density between the contents of the two vertical tubes. The difference of density is obtained by having a weak solution of uranine in one vertical tube, and pure water in the other.

In 1912* the authors studied the flow of distilled water along a capillary tube, the maximum rate of shear at the boundary

* Proc. Phys. Soc., Vol. XXIV., Part V., Aug. 15, 1912.

of the tube being about one radian per second. In the experiments to be described the rate of shear at the boundary of the tube was reduced to about one six-hundreth of a radian per second.

In 1912 the water was freed from air at the commencement of an experiment, but was exposed to air during the progress of the experiment, freedom from fungoid growths being ensured by the addition of a trace of copper salt. No evidence of abnormality was discovered in the coefficient of viscosity.

In the present experiment air-free water completely fills a closed circuit, and as there can be no absorption of air during the course of an experiment, it is not necessary to add a trace of copper salt to keep down the fungoid growths.

§2. Apparatus I.

(A) General Description of Apparatus.
(B) Theory of Method.
(C) Numerical Details of Apparatus.
(D) Difference of Density between Water and a Dilute Solution of Uranine.
(E) Details of Procedure.

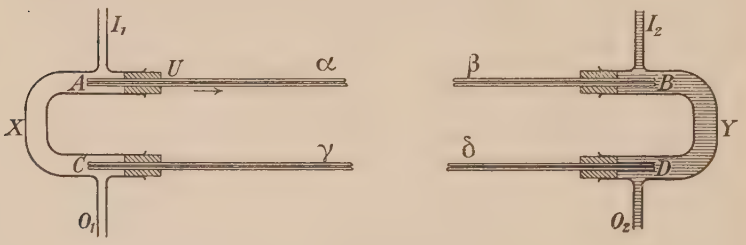


Fig. 1.

(A) *General Description of Apparatus.*—The apparatus is attached to a rectangular beam pierced at a distance equal to one-quarter its length from each end by two transverse arms at the ends of which work vertical screws supported on upright blocks. By rotating these screws the apparatus can be adjusted with considerable delicacy and the upper face of the beam made truly horizontal. Two straight capillary tubes, *AB*, *CD*, are fastened with hasps to the upper and lower faces of the beam so as to be parallel to one another, and, as indicated in Fig. 1, are connected so as to form a closed circuit by two U-tubes, *X* and *Y*, the joints being made with rubber

stoppers. The circuit is first filled with air-free distilled water, and the water in Y is then replaced by a dilute aqueous solution of uranine, the small driving force in the circuit being due, as already stated, to the difference in density between the solution and water. The solution of uranine is so dilute that to the degree of accuracy of the present experiments its viscosity may be assumed to be that of water. It may be noted that the present apparatus differs from that described in 1912 in that there is no surface of separation between air and water, and what is more important, there is no line of contact of air, water, and glass.

In the tube, AB, is inserted a short column, U, of dilute uranine solution which acts as an index whose linear movement measures the flow.

(B) *Theory of Apparatus I.*—

Let h=the vertical distance in centimetres between the centres of the capillary tubes, AB and CD.

δ=the difference between the densities of the liquids in X and Y in gms./cc.

g=the acceleration due to gravity in cms./sec.^{2}.

Then the driving head in dynes per square centimetre=$hg\delta$.

Let l_1=the length of the tube, AB, in centimetres.

$\frac{1}{r_1^4}$=the mean value of the fourth power of the reciprocal of the radius of the tube, AB.

Let l_2, $\frac{1}{r_2^4}$, l_3, $\frac{1}{r_3^4}$, l_4 and $\frac{1}{r_4^4}$ indicate corresponding values connected with the tubes, CD, X and Y.

Let η=coefficient of viscosity.

Q=rate of flow of liquid in cubic centimetres per second.

Then it can easily be seen that the head in dynes per square centimetre required to overcome the viscous resistance equals

$$\frac{8\eta Q}{\pi}\left(\frac{l_1}{r_1^4}+\frac{l_2}{r_2^4}+\frac{l_3}{r_3^4}+\frac{l_4}{r_4^4}\right).$$

Equating the two expressions for the head we obtain

$$hg\delta=\frac{8\eta Q}{\pi}\left(\frac{l_1}{r_1^4}+\frac{l_2}{r_2^4}+\frac{l_3}{r_3^4}+\frac{l_4}{r_4^4}\right).$$

The apparatus is of such dimensions that the last two terms within the brackets may be neglected.

Thus

$$\eta = \frac{\pi h g \delta}{8Q\left(\frac{l_1}{r_1^4} + \frac{l_2}{r_2^4}\right)}.$$

It may be mentioned that for ease of description the rectangular circuit is considered to be vertical, and in the experiments of this Paper this was the case. But, the tubes, AB and CD, retaining their horizontality, the circuit can be placed at any angle with the vertical. Let α be the angle the circuit makes with the vertical. Then h varies as $\cos\alpha$, and may have any value between zero and the distance separating the tubes, AB and CD. The same apparatus may thus be used over a wide range of rates of flow.

(C) *Numerical Details of Apparatus Used in Method I.*—

$$l_1 = 129{\cdot}2 \text{ cms.}$$

$$\frac{l_1}{r_1^4} = 4{\cdot}233 \times 10^{+6} \text{cms.}^{-3}.$$

$$l_2 = 129{\cdot}2 \text{ cms.}$$

$$\frac{l_2}{r_2^4} = 5{\cdot}449 \times 10^{+6} \text{cms.}^{-3}.$$

$$h = 5{\cdot}811 \text{ cms.}$$

(D) *Difference of Density between Water and a Dilute Solution of Uranine.*—The method employed to determine the difference of density was a slight modification of that employed by F. Kohlrausch.* Instead of using a silk fibre for the suspension, a platinum wire was employed, and this wire, where it entered the liquid, passed through a short glass tube, dipping into the liquid. Into this tube was poured a small quantity of pure paraffin. This arrangement avoided a platinum-air-water contact.

In the experiment for the determination of the viscosity the uranine solution in Y is not at exactly the same temperature as the water in X, and it is necessary to apply a correction for the difference of temperature.

Let e=the difference between the density of the uranine solution and that of the water. Let θ=the excess of the temperature of Y above that of X. Then, assuming that the coefficient of expansion of the dilute solution of uranine is not appreciably different from that of pure water, to the degree of

* F. Kohlrausch and Hallwachs, Wied. Ann., LIII., 1894, p. 1.

accuracy necessary in the experiments, in the neighbourhood 18·5-19·0°C., we may write

$$\delta = e - 0{\cdot}00020\theta.$$

It may be mentioned that the uranine used was the ordinary commercial product.

The graph in Fig. 2 gives the relation between density and concentration with one sample of commercial uranine. It

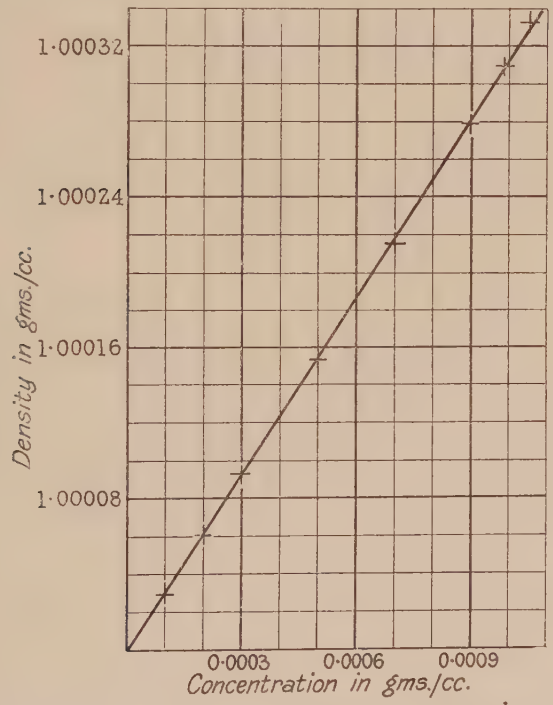


Fig. 2.—Variation of Density of Dilute Aqueous Solutions of Uranine with Concentration.

suggests that there is a linear relation between the density and the concentration.

(E) *Details of Procedure in Method I.*—The apparatus is not provided with taps, and the procedure in filling the apparatus is troublesome. In the first place, the authors designed an apparatus easy to fill and requiring taps of special construction; but in the conditions prevailing at the end of the war they could not obtain their requirements. The absence of taps has the advantage that there is no need to

consider the effect of any tap-lubricant on the viscosity of water.

It is necessary not only to fill Y with weak uranine solution, but also the neighbouring parts of the tubes AB and CD, otherwise the concentration of the solution in Y would vary with the flow. In addition, an index must be introduced in AB.

The inlet and outlet tubes (I_1, O_1, I_2, O_2) of the connecting tubes were fitted with stout caoutchouc pressure tubes, which were controlled by screw clips. (These details are not shown in the figure.) All the glass parts of the apparatus were carefully cleaned, and the capillary tubes were passed through indiarubber stoppers inserted in the ends of the U-shaped connecting ends. All rubber-glass joints were sealed externally with seccotine, and when this had dried the circuit was fastened to its supporting beam with hasps and filled with air-free distilled water.

Introduction of the Driving Column of Uranine Solution.—The apparatus was supported so that the two capillary tubes were in the same vertical plane, and I_1 and O_1, being closed, an aspirator containing an air-free solution of uranine in distilled water was connected to O_2, and 200 c.c. of this solution, whose concentration was 0·00097 grammes per cubic centimetre, were passed through the right-hand U-tube, the liquid emerging at I_2. The circuit was left in this position until the uranine solution had flowed along the tube CD through a distance equal to four-fifths of the length of the tube. The apparatus was then turned upside down, so that the flow took place in the opposite direction. When the length of the column of uranine solution in each capillary tube was about two-fifths of the length of the tube, the apparatus was placed on its side, the two capillary tubes being in the same horizontal plane.

The object of having columns of uranine solution in the right-hand side of each of the capillary tubes was to minimise any discontinuities at the ends B and D of the capillaries, and to ensure constancy in the value of the driving head. To make certain that the solution in Y was of the standard strength, an additional 250 c.c. were passed through after the apparatus had been allowed to rest for about 40 hours.

Introduction of the Recording Index into AB.—The apparatus was placed vertical, and, I_2 and O_2 being closed, the solution

of uranine was passed in at I_1, emerging at O_1. There were two paths for the solution—one through the connecting tube X and the other through AB, Y, and DC; but on account of the greater resistance of the latter path only a short column of the solution entered the left-hand side of the capillary AB. Air-free distilled water was then passed through from I_1 to O_1 until all the uranine in the U-tube X and in left-hand end of the tube CD had been removed, only the short index in the left-hand end of AB remaining. This required a large volume of water, and during its passage the coloured column of uranine was driven further into AB. I_1 and O_1 were then closed with screw clips.

Setting up of the Apparatus.—After the apparatus had been properly filled and the index introduced, it was allowed to rest for a week in a horizontal position. The apparatus was then taken into a thermostat room and supported as described in §2A, the screw feet being adjusted until the upper surface of the supporting beam was horizontal. To maintain a uniform and steady temperature around the U-tubes, each end of the apparatus was partly covered with a cylindrical sheet of iron standing on the bench. The top of each cylinder was covered with a sheet of paper, and the U-tubes completely surrounded and insulated with paper and cotton wool. A Beckmann thermometer was inserted in each cylinder, and so arranged that the end of the mercury column could be easily read.

Method of Reading Index and of Obtaining Volume Flow.—The method employed is similar to that made use of by the authors in earlier work.* An observer took three pairs of readings of the head and tail of the index U in the calibrated tube AB.

The mean of each pair was calculated, and then the mean of the three pairs taken as the reading of the observer. A numerical example will explain :—

Observer	I.			II.		
Reading of tail of index.....	8·7	8·9	8·9	10·0	8·5	9·2
Reading of head of index.....	19·7	19·2	19·2	18·4	19·5	18·8
Mean of these two readings ...	14·2	14·05	14·05	14·2	14·0	14·0
Mean reading of each observer	14·10			14·07		

The age of the index was 13 days.

* A. Griffiths, Proc. Phys. Soc., Vol. XXIV., Part V., August 15, 1912.

It will be noted that Observer II. had no definite points for the head and tail, but simply aimed at reading points where the intensity of the fluorescence was equal. As the index was symmetrical his method gave results consistent with themselves and with those of Observer I., who rather measured the points where the variation in the intensity of the fluorescence was most rapid. A scale drawn with Chinese white on black paper was placed behind AB, and the source of illumination was a wire-lamp fixed in a small portable box provided with a window of cobalt blue glass. These results gave the linear flow from which the volume flow was obtained by reference to the results of the calibration of this tube embodied in an appropriate correction curve.

§ 3. Apparatus II.

In the experiments performed with the first apparatus, the correction necessitated by the difference of temperature between the contents of tubes X and Y amounted to 20 per

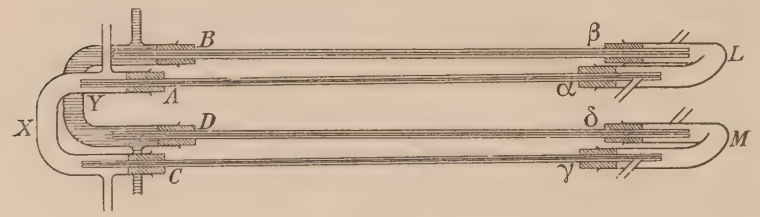


Fig. 3.

cent. of the head which would have existed in the absence of the variation in temperature. The need for this correction has been eliminated in a second apparatus in which the tubes X and Y are brought together, and both contained in the same insulating enclosure.

The form of the apparatus is the same as would be obtained by bending the capillary tubes AB and CD twice at right angles near their middles at α, β, γ, δ, so that the drying tubes X and Y and the portions of tubing $A\alpha$, $B\beta$ and $C\gamma$, $D\delta$ lie side by side, $A\alpha$, $C\gamma$ and $B\beta$, $D\delta$ being in parallel planes.

In reality the short portions of tubing $\alpha\beta$ and $\gamma\delta$ are replaced by drying tubes L and M, so that the apparatus actually consists of four parallel capillary tubes joined at their ends by four drying tubes to form one complete closed circuit (Fig. 3). It will be observed that as the drying tubes L and M are hori-

zontal, and X and Y are vertical, the head produced is independent of the temperature of L and M, and it is substantially independent of the temperatures of X and Y provided that these are the same, for we may assume the water and uranine solution to have the same coefficient of expansion within the narrow range of temperature covered by the experiments (18·5°C.-19°C.).

As in the previous experiments the apparatus is mounted on a wooden beam planed so that it is accurately rectangular.

The principle of the method, and the theory of the experiment are precisely the same as before, except that, we have to consider the resistance of four capillary tubes instead of two.

If l_A, l_B, l_C and l_D are the lengths of the tubes $A\alpha$, $B\beta$, $C\gamma$ and $D\delta$ respectively, and

$$\frac{1}{r_A^4}, \quad \frac{1}{r_B^4}, \quad \frac{1}{R_C^4}, \quad \frac{1}{R_D^4}$$

the means of the reciprocals of the fourth powers of the radii, since θ is zero the expression for η reduces to

$$\frac{\pi hgpe}{8Q\left(\frac{l_A}{r_A^4}+\frac{l_B}{r_B^4}+\frac{l_C}{r_C^4}+\frac{l_D}{r_D^4}\right)}.$$

Numerical details of Apparatus II.—

l_A=130 cm. $\quad \frac{l_A}{r_A^4}=1{\cdot}3252\times10^6$ cm^{-3}.

l_B=130 cm. $\quad \frac{l_B}{r_B^4}=1{\cdot}8703\times10^6$ cm.$^{-3}$.

l_C=130 cm. $\quad \frac{l_C}{r^4}=1{\cdot}7306\times10^6$ cm.$^{-3}$.

l_D=130 cm. $\quad \frac{l_D}{r_D^4}=1{\cdot}5271\times10^6$ cm.$^{-3}$.

h=5·752 cm.

The experiments were conducted just as before, the drying tube Y being filled with uranine, and an index U introduced into $A\alpha$ after filling the whole circuit with distilled evacuated water. In the course of three weeks the extreme variation of temperature at one point of the thermostat room was about 0·5°C. The extreme variation per day was about 0·2°C.

Apparatus	I.	II.			
Experiment	1	2	3	4	5
e (difference of density)	0·000406	0·000423	0·000423	0·0000423	0·0000423
θ in °C.	0·399	...	...	...	...
Total time of observation, in days	0·674	7·631	1·080	21·001	7·683
Total linear movement of index in cm.	24·73	283·41	39·33	75·81	27·85
Q in ccs./sec.	0·000007532	0·000013863	0·000013756	0·000001329	0·000001396
Mean temp. in °C.	18·67	18·89	18·83	19·02	18·67
η in gms./cm. sec.	0·010008	0·010403	0·010484	0·010850	0·010330
η calculated from Hosking's formula *	0·010394	0·010343	0·010357	0·010312	0·010394
Maximum rate of shear at boundary of index tube in radians/sec.	0·0233	0·0177	0·0176	0·00170	0·00179

* $\eta_t = \dfrac{0{\cdot}017928}{1+0{\cdot}03445t+0{\cdot}000235t^2}$ $(0 < t < 25°\text{C})$. (Phil. Mag., Vol. XVIII., p. 262, 1909.)

§ 4. Results and Conclusion.

The value of Q was found by assuming a linear relation between the volume-flow v and the time t. Then v=a constant+Qt, the constant being introduced to allow for the possibility of an error at the origin. The final evaluation of Q was made by the method of least squares.

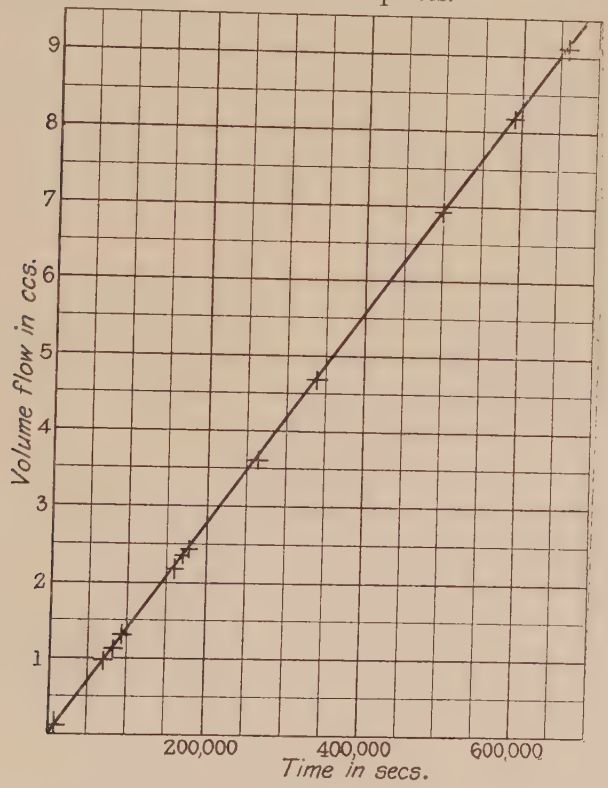


Fig. 4.—Apparatus II. Expt. 2.
Rate of shear=0·0177 rads./sec.

Fig. 4 shows the relation between the volume-flow and the time in the case of experiment (2)—*i.e.*, the first experiment with Apparatus II. In experiment 1 with apparatus 1 before applying the method of least squares it was necessary to allow for the variable difference of temperature between the two ends. The value of θ in the table is the average difference of temperature.

The discrepancy between Hosking's value of the coefficient of viscosity and the value determined in the experiment with the first apparatus is probably due to the difficulty in determining the difference of temperature between the two ends of the apparatus with sufficient precision. The fourth experiment was the first experiment to be performed at extremely low rates of shear, and in it the arrangements for obtaining uniformity of temperature at the ends of the apparatus were imperfect. These arrangements were improved in Experiment 5. Applying the temperature correction to the results recorded in the Paper the value of η at 18° C. is 0·010624 $\pm$0·000229 (assuming observations of equal weight), and 0·010569 from Hosking's formula. Thus, there is no experimental evidence that at the extremely low rates of shear (0·0017 to 0·0233 radians/sec.), the viscosity of air-free water in glass capillary tubes differs from its value at normal rates of shear (5,000-10,000 radians/sec.).

The authors wish to record their thanks for much valuable help to their assistant Mr. G. Temple, and to the Birkbeck College instrument maker, Mr. H. G. Bell.

DISCUSSION.

Dr. Vincent said it would be useful if the same apparatus could actually be used for high and low rates of shear. It would then be possible to find out any minor differences that existed better than by comparing results with different apparatus.

Dr. Rankine suggested that there was not much point in giving the results to so many figures. With regard to Dr. Vincent's suggestion it would probably be sufficient if the same capillaries were employed in conjunction with different driving apparatus for the different rates of flow, since it was in the capillaries that any errors were likely to arise. He thought, however, that the evidence of the Paper was sufficient to show that there is no difference in excess of 1 or 2 per cent. This result may, of course, only hold for water.

Mr. W. N. Bond said that for viscosities as high as those used by Davidson and Kurt Molin, the viscosity changed by 50 to 100 per cent. at low rates of shear.

Dr. W. Eccles said the authors were to be congratulated on having surmounted very considerable experimental difficulties, particularly those due to temperature variations. Mr. Bond's remark suggests that with more viscous substances the range of variable viscosity is reached earlier, and this suggests that the author's next experiments should be with oils.

Dr. Griffiths, in reply to Dr. Vincent, said that in their experiments of eight years ago, in which they had been dealing with low rates of shear, though not so low as in the present experiments, they had for comparison made measurements in which the water was driven through the apparatus at ordinary rates. The difference in viscosity was less than $\frac{1}{3}$ per cent. in that case. With the present apparatus there would be no difficulty in getting rates of flow a hundred times greater than those dealt with in the Paper.

XXI. *A Method of Measuring Frequencies.* *By* B. S. Smith *and* G. F. Partridge, *B.Sc.*

ABSTRACT.

An heterodyne method of measuring frequency by comparison with a calibrated valve oscillator is described. The calibration is performed by means of two valve oscillators capable of giving frequencies of, say, 1,000—/sec. upwards. The frequency of the oscillators is raised alternately to give a beat note whose pitch is determined by comparison with a fork. Intermediate frequencies are found by interpolation on the calibration curve and a method of checking the capacities of the oscillator condensers is described.

For the measurement of acoustic frequencies the sounds are converted into alternating currents by means of a suitable transmitter.

In experiments recently carried out by the authors it was found necessary to know, with a fair degree of accuracy, the frequency of sounds ranging from 2,000 vibrations per second to 10,000 vibrations per second. The usual methods of obtaining frequencies by comparison with standard forks are very uncertain at frequencies above 2,000 vibrations per second, since these depend on accurate tuning by ear of octaves, &c. The following heterodyne method has proved successful, and is available for acoustic or electric frequencies above 2,000 vibrations per second. It is also available for the determination of the frequency of damped oscillations, provided the damping is not too great.

The scheme is essentially an alternating current method and depends upon the conversion of the sound energy into electrical energy by means of a suitable form of sound receiver. In connection with this a valve oscillating circuit has been found a very constant form of alternating current generator, and a method is outlined for calibrating such a circuit for frequency by the same heterodyne method.

Reception of Sound.

For the reception of the sound an electrical receiver is required, which will not introduce other frequencies. A perfectly distortionless receiver is impossible, but it is usually sufficient to select a receiver whose resonant point is well above or below the frequency to be measured. The type of receiver also depends upon the sensitivity required

and the conditions under which the sound is given out. If the sound is travelling through the air the sound waves can be made to act directly on the diaphragm of the receiver; if travelling through water, one of the well-known hydrophones can be used. In most cases either a telephone earpiece or a microphone transmitter connected through a transformer can be used. It is very necessary to ensure that the transformer does not introduce other frequencies.

Heterodyne Method.

The principle involved is the interference produced between two alternating currents when introduced into the same detector circuit. For the latter, either a valve or a crystal can be used (*see* Fig. 1, showing the circuit for a crystal detector). The function of the detector is to rectify both the alternating currents, so that the best note can be clearly

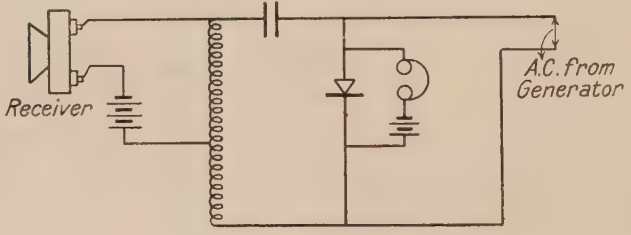


Fig. 1.

heard, while the higher frequencies are practically eliminated.

The alternating current coming from the sound receiver or from the circuit whose frequency is to be measured, is therefore introduced into the rectifying circuit together with the alternating current of about equal magnitude from the generator, the frequency of the latter being adjusted to a frequency greater than that of the sound under measurement by, say, 1,000 vibrations per second. This frequency is then carefully adjusted until the beat note heard is exactly in unison with a 1,000 fork. The frequency of the alternating current given by the generator is known from the calibration curve for the generating set, and hence the frequency of the sound in question.

Alternating Current Generator.

The oscillating valve circuit employed is that with variable capacity and fixed inductive coupling between the plate and grid circuits (*see* Fig. 2). The particular valve used, a Type Moyenne valve, French make, working with a filament voltage of 6 volts and a plate voltage of about 200 volts, enables all frequencies between 300 and 30,000 cycles per second to be obtained by variations of the capacity in the oscillating circuit from about 1·7 microfarads to zero.

Providing the coupling between the oscillating circuit and the output winding is sufficiently loose, no difficulty is

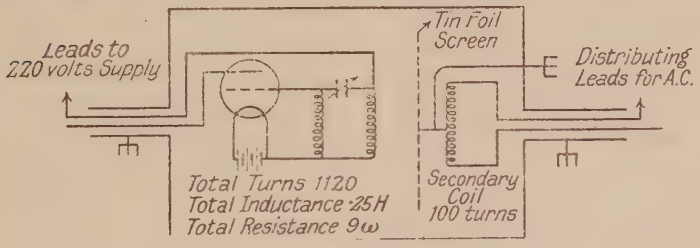


FIG. 2.—PORTABLE OSCILLATING SET.

experienced in maintaining a constant frequency, and for most purposes a 200-volt lighting circuit may be used in place of a high tension battery.

Calibration of the Generator.

Since the frequency of the alternating current is adjusted by varying the capacity in the oscillating circuit, a curve connecting capacity and frequency is all that is necessary. This can be obtained as follows by the heterodyne method, a second alternating current generator being required for comparison. For clearness we will denote the set to be calibrated as *A* set and the comparison set as *B* set.

The alternating current from both sets is brought into a crystal detector connected through telephones. *A* is made to oscillate exactly at 1,000 cycles per second by comparison with a 1,000 fork, using the beat method without a rectifier. *B* is then raised in frequency, keeping *A* constant, until a beat of 1,000 is heard through the detector telephones, the beat note of 1,000 being adjusted exactly to unison with

the 1,000 fork. The frequency of B is now 2,000 and is kept constant, while that of A is raised until the beat note disappears, then comes in again and rises to 1,000, to which it is accurately adjusted. The frequency of A is now 3,000. By repeating the process frequencies of 3,000, 5,000, 7,000, &c., cycles per second are obtained in the A set, and the

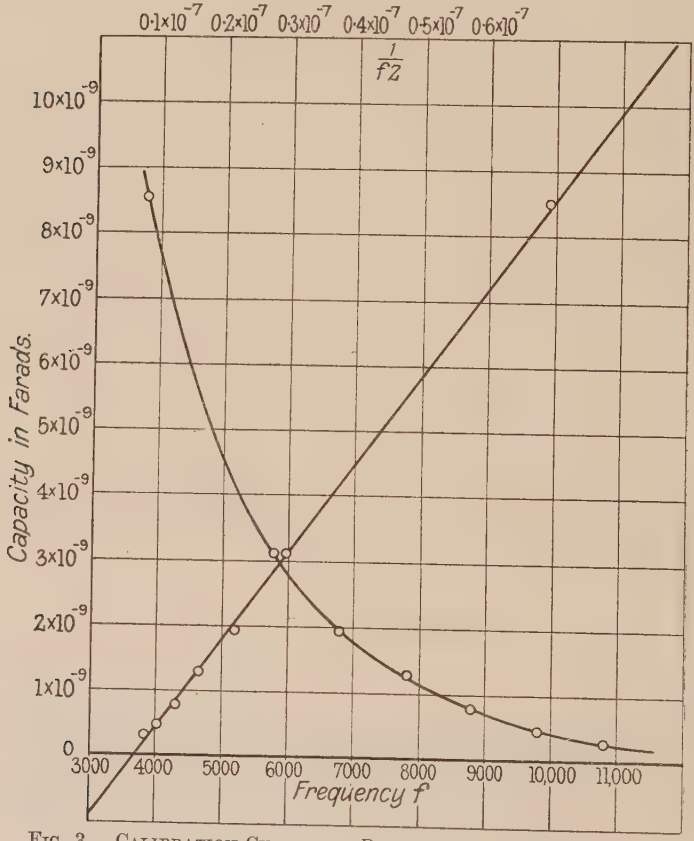


FIG. 3.—CALIBRATION CURVE FOR PORTABLE OSCILLATING SET.

required capacity noted in each case. Other series of frequencies commencing, say, at 1,500 and 2,000, are then obtained by the same process, the former requiring the use of a 500 fork.

The calibration curve giving frequency against capacity for the generating set described above is given in Fig. 3,

and the graph connecting the reciprocal of the square of the frequency with capacity is practically a straight line.

Calibration of Condensers.

In order to use the curve for frequencies intermediate to those measured it is not essential to know the absolute value of the capacity, but it may be necessary to calibrate the condensers in terms of the vane condenser. For example, if plug condensers giving $\frac{1}{2}$, 1, 2, 4, 8, &c., millimicrofarads and a vane condenser with a maximum capacity of $1\frac{1}{2}$ millimicrofarads are used, the same frequency can be obtained usually with two or three different settings of the plug and vane condenser. In this way, starting at the highest frequency the values of the plug condenser can be found in terms of the vane condenser. The law of the vane condenser can be obtained by a bridge method, and the calibration curve of the generator, connecting frequency and capacity, may be made in terms of the vane condenser.

DISCUSSION.

Dr. VINCENT said he would like the authors to give more detail about the coupling of the middle portion of the apparatus with the end parts. The authors appeared to have reached the same conclusion as himself that a triple arrangement was most suitable for measuring frequency differences. Dr. Eccles and he had called attention to a somewhat similar apparatus which they had termed a differential wavemeter. The two outer parts generated the oscillation and the middle part resonates sufficiently to either of them that you could do anything you liked to the middle circuit without affecting the capacity, &c. With the old arrangements it was necessary for the observer to stand in a particular position and to abstain from making any movement whatever. He had only one point of criticism. He thought it was undesirable to get the electricity wholesale. A different supply should be used for the two circuits. It was important to use large batteries which could be left on for some time to give steady conditions. With such precautions he thought it would be possible to get an accuracy of 3 or 4 in 10 million.

Dr. D. OWEN asked to what purposes the authors had actually applied their method. Could it be used for calibrating a Galton whistle; would the sounds in this case be sufficiently intense for the method to be applicable? He presumed the authors had determined by it the limit of audibility. This was usually stated to be 40,000 vibrations per second, but he had found people who were deaf above 10,000. Could the method be used to study the overtones of a valve set?

Prof. RANKINE asked if the authors could actually produce a note of 10,000 frequency.

Prof. ECCLES said there was often some diffidence in confessing to deafness to high notes as the limit of audibility was a function of age.

Dr. RAYNER said he presumed the work had been done before the Abraham Block multi-vibrator was described.

Mr. PARTRIDGE, in reply to the discussion, said he had seen Dr. Vincent's work, but thought he was principally concerned with the measurement of

inductances. As regards coupling, the middle circuit contained the telephone and crystal rectifier in series with two small secondary coils associated with the inductances of the outer circuits; there was no tuning. As regards the electric supply, their reason for using the 220-volt lighting circuit was because of its convenience on board ship. In reply to Dr. Owen, they had found the method very useful in determining the frequency of heavily damped steel diaphragms. It would be possible to use it with a Galton whistle if some means of picking up the sound at such high frequencies was available. In reply to Dr. Rayner, he was unaware of the device he had referred to.

(Mr. Partridge then demonstrated the production of a note of 10,000 frequency. This appeared quite loud to many of the audience, but was inaudible to others.)

XXII. *Note on Vacuum Tubes Used as Detectors of Electrical Oscillations.* *By* L. Hartshorn, *D.I.C., A.R.C.S., B.Sc., and* E. S. Keeping, *D.I.C., A.R.C.S., B.Sc.*

Received January 21, 1921.

ABSTRACT.

I. The Paper describes the development of a robust form of vacuum tube which was used as a detector of electrical oscillations in the "wireless" circuits carried by aeroplanes. Platinum electrodes are avoided being replaced by strips of tinfoil, to which contact may be made by the spring clips holding the tube in position.

II. It was found that when a discharge is passed through such a tube, the walls are affected in such a way that thenceforth it is much easier to get a discharge to pass. The change produced by the first discharge s annulled by heating the tube above 210°C. Further, if the walls are coated on the inside with a metallic film, this first discharge is unnecessary, and the tube is unaffected by heating, but when the walls are coated with an insulator it is if anything more difficult to pass a discharge. A silica tube behaves like one coated with metal. It seems possible that the change in the tube may be due to the formation of a layer of gas molecules on the walls by the first discharge. The explanation of the behaviour of the silica tube is a difficulty.

I. The Design of the Tubes.

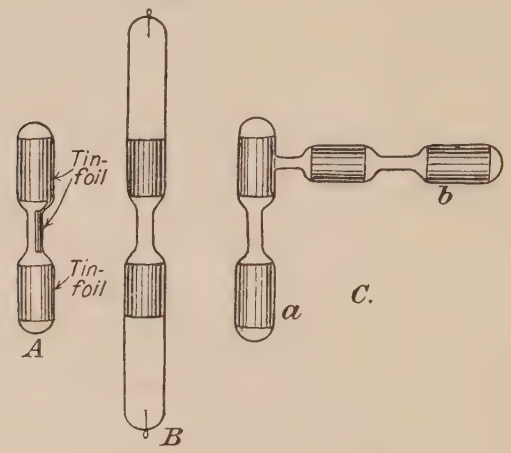
This work was carried out during the War, and arose out of the demand for vacuum tubes which could be used as indicators of tuning for the circuits forming the wireless equipment of aeroplanes. The tubes were not to be more than 7 cm. long, they were to be as robust as possible, and since this form of detector is rather crude, as sensitive and as easily seen as possible.

Two tuned circuits were set up. The primary consisted of an inductance, a condenser and spark gap, and was energised by an induction coil. The secondary consisted of an inductance and variable air condenser, and was magnetically coupled to the primary. The vacuum tube was connected across the condenser of the secondary circuit, and of course should glow when resonance is established.

Preliminary Experiments.—As a preliminary several old vacuum tubes which happened to be available were tried. Some would glow under the above conditions, but others would not. The best results were obtained with a tube of krypton. It was then noticed that a tube which would not glow when supported in a wooden clamp would glow when held in the hand. This suggested wrapping the tubes round

with tinfoil, which was earthed. A further improvement was obtained by connecting the tinfoil to one electrode instead of to earth.

A number of experimental tubes were made. These were of the form commonly used for spectroscopy. The diameters of the narrow portion varied from 1 mm. to 8 mm. They were all 7 cm. long, and had platinum-wire electrodes. They were connected to a vacuum pump, McLeod gauge, and drying tubes, and were in turn filled with air, carbon dioxide, and hydrogen at pressures varying from 3 mm. to ½ mm., but no glow could be obtained with or without the tinfoil.



A—Final form of tube detector.

B—The tube was made "soft" by a discharge passed either by way of the platinum wire electrodes or the tin foil.

C—"*a*" was made "soft" by a discharge in "*b*."

Experiments with Helium.—It was to be expected that helium and neon would give the best results. Accordingly helium was introduced into the apparatus, and the tubes again tested. In the process of filling, a discharge was passed through the tubes directly from the induction coil, in order to observe the progress of purification of the gas by examination of the spectrum.

After this treatment all the tubes would glow in the tuned secondary circuit. The results may be summarised thus :—

(1) The best pressure of gas was 15 to 20 mm.

(2) The best diameter of the narrow portion of the tube was 2 to 3 mm.

(3) None of the tubes would glow using the electrodes only. In all cases it was necessary to coat with tinfoil the middle as well as the two ends of the tube, and the best results were obtained by connecting the tinfoil to one electrode.

Experiments with Neon.—A new set of tubes was made and filled with neon. None of these would glow on the secondary circuit until a discharge had been passed through them from the induction coil. After this treatment they glowed brightly. The following points were noted :

(1) The wider tubes would glow without being coated with tinfoil, but the narrower ones would only glow when the foil was on, though it was not necessary to connect it to an electrode.
(2) Connecting the tinfoil to one electrode considerably improved the glow with the narrowest tubes, but made little difference in the other cases.
(3) The glow obtained when the tinfoil was used as one electrode was as good as that with both electrodes and the tinfoil.
(4) Using the tubes in this way the glow was not increased by raising the pressure above 10 mm., but at 6 mm. pressure the narrower tubes would not glow steadily, and the wider tubes were not as brilliant as at the higher pressures.
(5) At 11 mm. pressure a tube with a capillary portion of 2 mm. diameter was most satisfactory, but at 6 mm. pressure a diameter of 3 mm. was better.
(6) The glow was brighter than that given by the helium tubes.

In order to make the tubes as robust as possible, and in order to facilitate their construction, it was resolved to try to work without electrodes, using instead two pieces of tinfoil on the outside of the tube. This was found to be satisfactory, but it was found that the glow was more steady if the capillary was also coated with tinfoil connected to one of the end pieces, a gap of about 5 mm. being left between the two pieces of foil. The widest tube would not glow under these conditions, but the rest would ; indeed, better results were obtained in this way than by the other. Tubes with the narrow portion 1 to 2 mm. diameter were found to be most satisfactory. Using the tubes in this way a brilliant glow could be obtained at a

pressure as low as 3 mm. A pressure of about 5 mm. seemed to give the best results.

Further tests showed that the platinum-wire electrodes were not at all necessary, and the form of tube finally adopted is shown at (*A*). The diameter of the narrow portion was 1 mm., and the gas pressure 5 mm. Before filling, all the tubes were strongly heated (a vacuum of something like 0·01 mm. being maintained) until they were on the point of softening. None of these tubes would glow on the secondary circuit after filling, until a discharge had been passed through them directly from the induction coil. Very few of the tubes would glow at once, even when connected to the induction coil; there was sometimes a lag of more than a minute, but after this treatment the tubes glowed without any lag on the secondary circuit. This last condition appeared to be fairly permanent, as tubes kept for two months glowed without any lag.

On trying air in tubes of this pattern it was found to glow under the same conditions as the neon, but the glow was not nearly as bright as that obtained with neon, and was unsatisfactory because there was often a considerable lag between the application of the voltage and the appearance of the glow, this lag gradually increasing with age.

II. Phenomena Occurring in the Tubes.

The above phenomena invite further investigation. These tubes when first made would not glow on the oscillatory circuit. For convenience such tubes were referred to as "hard." Even when directly connected to the induction coil they did not glow at once, but did so after a lag varying from a few seconds to a minute. When, however, this discharge had passed, the tubes would glow on the oscillatory circuit and without any lag. This condition was referred to as "soft" and seemed to be permanent.

Phenomenon Associated with the Walls of the Tube.—It was first necessary to find out whether the change in condition was due to change in the gas, or in the walls of the tube. For this purpose two tubes were made and connected by a piece of glass tubing 30 cm. or 40 cm. long. This arrangement was filled with neon at 4 mm. pressure. At first, of course, neither of the tubes would glow on the oscillatory circuit (both tubes were "hard"). A discharge from the induction coil was now passed through one tube and then it would glow on the oscillatory circuit (it was "soft"). If the softness is due to a

change in the gas, then when mixing of the gases in the two tubes occurs by diffusion, we should expect the other tube to be soft also. However, such was not the case ; the other tube remained "hard." Secondly, a tube of neon was rendered "soft" and the gas pumped out. Fresh neon was then admitted. The tube was still "soft." Hence the effect is associated with the walls of the tube.

Phenomenon not Confined to Neon.—It was found that the phenomenon was shown quite well by tubes containing air. This considerably simplified the experimental work, as the quantity of neon available was not large. The results hereafter mentioned refer always to tubes containing air unless the contrary is stated. It was found that a tube which had been rendered "soft" by a discharge through neon in it, remained so when the neon was replaced by air, even though in the meantime the tube had been standing for a day or two with full atmospheric pressure inside it, and in the same way a "hard" tube rendered "soft" in air, continued to be "soft" for neon.

Effect of Temperature.—In the preparation of the vacuum tubes, to be used as detectors, it was customary to heat the tubes before filling as strongly as possible without risking collapse, the pressure inside being kept as low as possible by means of the Gaede pump. In spite of this treatment further heating after sealing off a tube was found to spoil it, the glow obtained being a pale blue instead of the reddish neon glow. If, however, the tube was heated while still joined to the filling apparatus, after having been rendered "soft" by a discharge from the coil, the only effect was to render the tube "hard." This effect of heating was the same whether the tube contained air or neon, and whether the heating was conducted with the pressure inside very low or atmospheric. A tube not heated before filling was hard like the others, but this, of course, was only to be expected in view of the heating in process of manufacture.

It was thought worth while to investigate further this heating effect. A soft tube was heated to 100°C. in boiling water for some time, with apparently no effect. Similarly no change was observed when a tube was heated to 200°C. in a bath of oil. The tube was then heated in a small oven made of asbestos board and heated by passing an electric current through a coil of nickel wire. A tube, originally soft, after having been maintained at a temperature of 257°C. for

about 20 minutes was found to be hard. A soft tube heated to a temperature of 205° for abour an hour was still soft, but when heated to 212° for about an hour, it was made hard. These results were confirmed several times. Extreme accuracy was not aimed at in the measurement of temperature, but we may safely say that the change takes place at about 210°C.

Effect of Nature of Walls of Tube.—(*a*) A little sodium was distilled into one of the tubes so as to form a film covering the walls but leaving the capillary clear. This tube was then filled with neon. It was found to glow at once on the oscillatory circuit.

(*b*) The walls of another tube were coated with a film of platinum. This was obtained by painting the walls with "liquid platinum" and then heating strongly. This tube (when filled with air) would also glow at once on the oscillatory circuit and it was not made "hard" by strongly heating, either when the atmosphere was in the tube, or the tube was evacuated.

Tubes whose walls were coated with various other substances were tested. In general it was found that when the covering was a conductor the tubes were "soft" and were unaffected by heating. When the covering was an insulator the tube was "hard." In the case of paraffin wax and sealing wax, the tube remained hard even after passing a discharge direct from the induction coil, but in the case of sulphur such a discharge made the tube "soft" exactly as for uncovered glass walls. When the sulphur was melted and run over the walls afresh the tube again became hard.

One tube was made of silica. This was found to be soft when first made, and it was found to be still soft even after making it red hot in the blowpipe flame.

The presence of a drop of mercury in a tube made it soft, and it was noticed that the discharge not only passed between the two pieces of tinfoil but also down to the mercury.

The first discharge, which was passed so as to make the tubes soft, was at first always passed by way of the tinfoil so that it passed through the walls of the tube. It was obviously of interest to find out whether it was necessary that it should pass thus or whether a discharge through electrodes had the same effect. Accordingly, a tube was made furnished with both electrodes and tinfoils, the two being some distance apart (*see* Fig. B). At first, of course, the tube would not glow on the oscillatory circuit, but it would do so after a discharge

had been passed through the tube using the electrodes, and the presence or absence of the tinfoil whilst this discharge was passing made no difference. Thus a discharge through the electrodes has the same effect as a discharge which passes through the glass.

In order to find out whether the mere presence of the electrostatic field was sufficient to produce the change or whether it was necessary that a discharge should pass, the tube was evacuated as completely as possible, and then connected to the induction coil. In some cases the tubes were made soft, and in others not so. It was probable that in the former cases a discharge had passed, but the results were not conclusive.

As an alternative the atmosphere was admitted into the tubes, and they were then connected to the coil. In no case did this treatment make a tube soft.

It was further discovered that the effect of this discharge in rendering a tube soft was not only exerted on the glass immediately beneath the tinfoils, but also on that some distance away. Two tubes were joined by a cross-piece as at *C* (*see* Fig.). Tube *b* was made soft by a discharge. The room being darkened it was observed that the glow extended along the cross-piece as far as tube *a*. On testing *a* it was found to be soft.

These experiments seem to suggest that the effect of this discharge is due to the bombardment of the walls of the tube by the ions taking part in the discharge.

Measurements of the Discharging Voltages.—It was now resolved to measure the voltage required to make a tube glow, as it seemed possible that such measurements might give more information than merely knowing whether a tube was hard or soft. The induction coil was used as a transformer and fed by alternating current. The tubes were connected across the secondary circuit. By means of a rheostat in the primary circuit the voltage across the secondary circuit, *i.e.*, across the tubes, could be varied. The R.M.S. values of the voltages were measured by means of a multicellular electrostatic voltmeter.

The glow produced by this means was not nearly so brilliant as that produced by the previous arrangement. This is probably due to the difference of wave form. Probably in the first case the curve had sharp peaks whilst in the second it was approximately a sine curve. Further the phenomenon

was not nearly so well marked in these experiments as might have been expected, *e.g.*, the average value of the voltage required to send a discharge through a " hard " tube containing air was 480, while for a soft tube it was about 390. In some cases a hard tube would glow at 450, while some soft tubes required about 410. Complications also arose from the fact that the discharge was not always of the same character. Sometimes a peculiar glow first appeared extending only half way down the tube, but as the voltage was gradually increased, the usual full glow more or less suddenly appeared. In some cases no glow whatever appeared until an unusually high voltage was reached, and then the ordinary full glow would come quite suddenly.

The neon tubes seemed to be the most consistent in their behaviour. They all first lit up with the faint glow only extending part of the way along the tube. For a soft tube, this glow appeared at 260–290 volts, whilst the full glow appeared at 360–400. For a hard tube, the first glow appeared at about 330 and the full glow at about 415.

With the air tubes it was exceedingly difficult to get consustent results. A new tube would generally light up at 460–500 volts. A tube which had just been made soft would generally glow at about 380, and if such a tube were heated and left for some time it would once more require 460–500 volts, but if it were tried *immediately* after heating, it seemed as though a lower value was required than if left for an interval. It seems possible that the various values obtained for air tubes corresponds to the varying lags obtained in previous experiments, though it should be mentioned that in making the measurements the voltage was increased very slowly to allow for the lag.

Conclusion.

In these tubes with external electrodes the discharge has to pass through the walls of the tube, hence it must pass as a " condenser " current, and the discharge must be alternating. This was strikingly shown by connecting a tube across a spark gap, excited from a high-tension direct-current generator. The tube only glowed when a spark passed across the gap, *i.e.*, when oscillations were occurring.

Thus we may regard the pieces of tinfoil on the tube as the outer plates of two condensers, the walls of the tube being the dielectric. The inner plates of these condensers must be the inner surface of the tube, and it seems possible that a

" soft " tube is one in which there is a conducting layer of gas molecules sticking to the walls, and that it is the function of the first discharge to form this layer. Heating might remove these molecules, and thus make the tube hard. This idea, however, leads to no simple explanation of the behaviour of the silica tube.

The recent publication of Papers* on the disappearance of gas in the electric discharge has drawn our attention to this phenomenon, and suggested that the effects observed in our experiments are intimately connected with it. It seems quite probable that the first discharge passed through our tubes was accompanied by the " disappearance " of some of the gas which became attached to the walls in some manner, and modified the glass-air surface so as to facilitate the discharge.

In conclusion we wish to thank the Rt. Hon. Lord Rayleigh for much valuable advice, and for the keen interest he showed in the work during its progress in his laboratory.

DISCUSSION.

Prof. G. W. O. Howe mentioned that comparisons of vacuum tube detectors both with internal and external electrodes were carried out at the Reichsanstalt. Helium at various pressures was employed. Certain pressures were used to obtain extreme sensitiveness, while others were used if extreme brightness was desired. In some of the tubes issued internal electrodes were fitted, not for actual use, but for the initial preparation of the tube by electrolytic deposition of potassium on the walls.

Dr. Vincent hoped the authors would not cease work on this interesting problem with the publication of the present Paper. They had dealt only with one aspect so far, but there were a number of interesting researches, particularly in connection with valve-maintained circuits, which would suggest themselves. Mr. Hartshorn had mentioned the advantage possessed by the method that it responded without lag during quick tuning. Another advantage was that there was no rectification of the current.

Mr. G. D. West asked if many of the authors' results in connection with the initial hardness of the tubes and the permanent softening effect of an initial discharge could not be accounted for by minute perforations of the walls by the discharge. The effects of subsequent heating appeared to fit in with this hypothesis.

Mr. W. R. Cooper asked whether the authors had looked up the work of Claude on neon tubes. These were large tubes for illumination, and it was found that a tube 6 metres long required less than 800 volts. Possibly some of the effects observed by the authors (*e.g.*, on heating the tubes) were due to the fact that the slightest impurity produced a marked effect. Claude found that it was not sufficient to use pure neon; he therefore purified the neon when in the tube itself by connecting the tube to a receiver containing carbon and immersed in liquid air.

Dr. Borns asked if a film of moisture might give rise to any of the effects.

A Visitor said that a common form of indicator in use for tuning purposes was an ordinary flash-lamp bulb. What were the conditions necessitating

* " Phil. Mag.," November, 1920. This Paper gives references to the earlier work.

the use of vacuum tubes? He noticed that when responding to spark discharge it was difficult to note the brightest point on account of the intermittence of the light. This flicker is absent with a flash lamp. The response of such bulbs was also quite quick enough for all ordinary conditions. Would a fluorescent screen in the tube increase its efficiency? What was the effect of the frequency of the oscillations on the voltage necessary for discharge?

Mr. HARTSHORN, in reply, said he was not familiar with all the early work which had been mentioned. They had assumed that the experts who had referred the problem to them were aware of all that had been done on it. He did not think perforations would explain the softening of the tubes, as this would not fit in with the behaviour of some of the compound tubes. As regards moisture, the greatest care was always taken to exclude this. As regards the use of vacuum tubes it was essential that the signals should be easily seen on an aeroplane, and flash lamp bulbs required more careful watching. As regards the intermittence, this was certainly a drawback, but was only present with spark discharges. It was not there with valve circuits. A fluorescent screen would make the tubes much too difficult to construct. Something very simple and robust was required. He did not know the effect of frequency on the discharge voltage. The figure he gave, 260 volts, was at ordinary supply frequency.

XXIII. *On the Coefficient of Diffusion of Certain Saturated Solutions.*

By Basil W. Clack, *M.Sc. (Lond.), Lecturer in Physics at Birkbeck College.*

Received February 23, 1921.

ABSTRACT.

This Paper gives an account of experiments on the Diffusivity of saturated solutions of KCl, NaCl and KNO_3 at constant temperatures near 18°C., when the steady state of diffusion has been attained, employing a method similar to that previously used by the author (Proc. Phys. Soc., Vol. XXI., p. 863, 1908; Vol. XXIV., p. 40, 1911; Vol. XXVII., p. 56, 1914; Vol. XXIX., p. 49, 1916).

The solution under investigation is maintained at complete saturation by the presence of salt crystals in the diffusion vessel. The theory takes into account the change in volume of this salt as it dissolves, and an expression is obtained for the coefficient of diffusion at complete saturation, which depends on the rate of change in weight of the diffusion vessel with time. The experimental results are found to agree very closely with the values obtained by extrapolation from the results previously found for less concentrated solutions. By the present Paper the author has thus extended the range of concentration over which he has studied diffusion from very dilute solutions right up to complete saturation.

I. *Introduction.*

In the study of the diffusion of salts through water, not only have investigators very rarely employed steady conditions, in which the concentration of the solution remains constant at each point of the diffusion tube all the time the experimental readings are being taken, but in the majority of instances the solutions under investigation have been comparatively dilute, and very few experiments have been undertaken in which relatively concentrated solutions are employed.

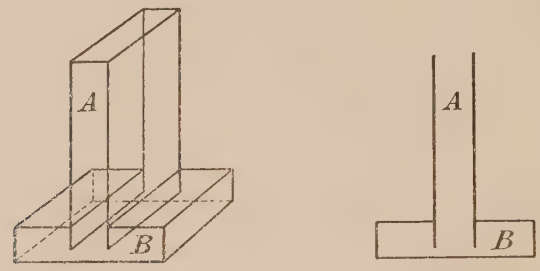
The author has already insisted on the importance of adopting the steady state in these researches (*see* Proc. Phys. Soc., Vol. XXI., p. 863, 1908; Vol. XXIV., p. 40, 1911; Vol. XXVII., p. 56, 1914; Vol. XXIX., p. 49, 1916), and in the present Paper a method is described in which this condition is employed in the study of saturated solutions.

II. *Apparatus.*

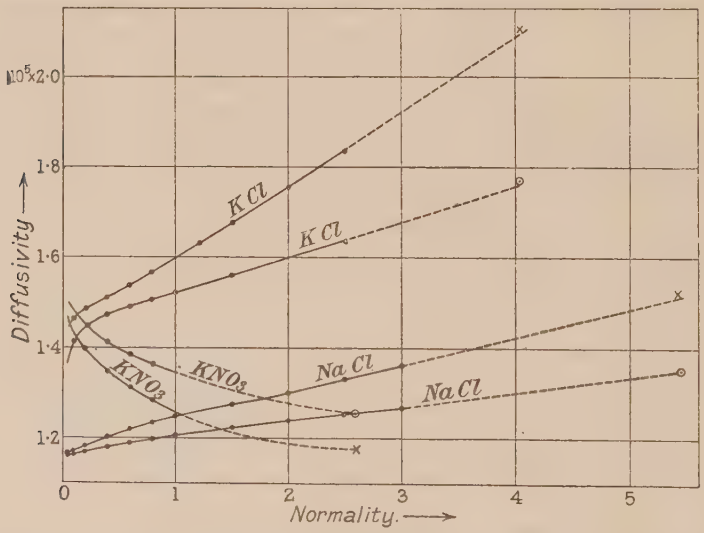
The apparatus contains a diffusion tube, *A*, which has been made rectangular in cross-section in order to make the same apparatus applicable to another investigation which the author hopes to carry out in the future. The tube, which has a width of about 4 cm. and a breadth of 1 cm., and is

5 cm. high, is made of glass plates cemented together with zinc oxide and water glass cement, and is fitted at its lower end into a shallow glass box, *B*, about 4 cm. square and 1 cm. high, as shown diagrammatically in the figure.

The whole of this apparatus is filled with air-free saturated



solution, and the concentration of the solution at the lower end of the rectangular diffusion tube is maintained constantly at complete saturation by filling the shallow box with crystals of the salt under investigation. The diffusion cell thus filled



is suspended from one arm of a delicate balance by a fine platinum wire, gauge No. 33, and hangs a few centimetres below the surface in a large tank containing about 30 litres of distilled water maintained at a constant temperature of about 18°C. in a thermostat room.

After diffusion has been proceeding for a sufficient time, the concentration of the solution at the upper end of the diffusion tube may be taken as zero on account of the very large volume of water present in the tank. Thus the concentration of the solution in the diffusion tube varies from saturation at its lower end to zero at its upper end, and at every point remains constant with time—*i.e.*, the steady state has been established. When this condition has been obtained, the rate at which salt diffuses out of the cell has become constant, and observations of the weight of the cell every day enables this rate of diffusion to be determined. As the weight changes in these experiments by about 1 mg. in eight or nine minutes, it is only necessary to adjust the mass in the other pan of the balance to the nearest milligramme less than that of the cell, and wait until the pointer is seen to move to the zero of the micrometer scale fitted in the eyepiece of the observing microscope. The usual precautions already described are taken to avoid disturbance of the solution during these weighings.

As the diffusion cell has so small a volume, any variation of the density of the water in the bath, caused by slight variations in temperature, or by the addition of diffused salt to it, only produces a negligible effect on the weight observed, and hence it is not necessary in these experiments to employ a compensating cell suspended from the other arm of the balance.

III. *Theory.*

The theory of the method outlined above is slightly complicated by the change in volume of the salt crystals as they dissolve. It is known that on solution a decrease in volume usually occurs, so that water must enter the cell from the tank to make up for this change in volume, and the diffusion of the salt out of the cell takes place simultaneously with the entrance of water into it.

If v_0=velocity downwards with which the water enters the top of the diffusion tube, and if

L=total length of the diffusion tube in centimetres,
A=its area of cross-section in square centimetres,
N=concentration of saturated salt solution in gms./c.c.
D=density of saturated salt solution in gms./c.c.
c=mass of salt diffusing out of the cell/sec. when the steady state has been reached,
i=change in weight of cell/sec. when the steady state has been reached,

then it has been shown by the author (Proc. Phys. Soc., Vol. XXIX., pp. 51, 52, 1916) that

$$K=Li/\left\{AN\left[1-\frac{v_0N}{2c}+\frac{v_0(v_0-bc)N^2}{3c^2}\right](1-\delta)\right\},$$

where it has been assumed that the relationship between the density and the concentration of the solution, which is approximately a linear one, may be expressed with sufficient accuracy as $d=1+an$. Thus a is a constant whose value may be taken as equal to the mean slope of the true curve connecting d and n and b is written for $(1-a)$.

In previous work it was found unnecessary to include the third term in the square bracket, since (v_0-bc) was so small that this term was negligible, but in the present case rather different conditions result from the solution of the salt crystals in the reservoir, and this has the effect of slightly increasing the value of the term under discussion.

We have seen that as salt leaves the cell by diffusion, water enters it. Let δ=the ratio of the mass of water entering the cell to the mass of salt leaving it in a given time when the steady state has been attained.

Then δ can be calculated as follows :—

Let N=concentration of saturated salt solution in gms/c.c.
D=density of saturated salt solution in gms/c.c.
Z=density of the salt crystals.
V_1=volume of solution in the cell of any given instant.
v_1=volume of salt in the cell of any given instant.
V_2,v_2 corresponding quantities at a later instant.

Then loss of salt in this time interval

$$=V_1N+v_1Z-V_2N-v_2Z$$
$$=(v_1-v_2)Z-(V_2-V_1)N.$$

Again, gain of water in the interval

$$=(V_2-V_1)(D-N)$$

Now

$$\delta=\frac{\text{gain of water}}{\text{loss of salt}}=\frac{(V_2-V_1)\,(D-N)}{(v_1-v_2)Z-(V_2-V_1)N}.$$

But $V_1+v_1=V_2+v_2$=total volume of cell,

i.e., $V_2-V_1=v_1-v_2$,

i.e., $\delta=(D-N)/(Z-N)$.

From published tables of densities the value of δ can thus

be determined. For the salts investigated in this Paper its value is in the neighbourhood of 0·5.

Again it is clear that in the steady state the mass of water entering the cell per second=(mass of salt leaving it per second) δ ; that is

$$v_0 = c\delta.$$

Hence we have

$$K = Li/\{AN[1-\tfrac{1}{2}N\delta+\tfrac{1}{3}N^2\delta\,(\delta-b)]\,(1-\delta)\}.$$

The difference between δ and b is, even in the present work, so small that the omission of the third term in the square bracket only produces an error in K which amounts in all cases to a fraction of 1 per cent.

IV. *Results.*

The results obtained by this method, using saturated solutions of the salts mentioned, are collected and tabulated below :—

Salt.	KCl.	NaCl.	KNO_3.
Temperature °C. ...	17·3	17·1	18·1
Normality	4·03	5·44	2·575
N.gms/cc.	0·300	0·318	0·260
D.gms/cc.	1·178	1·202	1·154
Z.gms/cc.	1·977	2·150	2·092
δ	0·5236	0·4827	0·4878
b	0·4059	0·3640	0·4080
L cms.	5·04	5·04	5·04
A sq. cms.	4·22	4·22	4·155
i gm/day	0·1687	0·1486	0·1115
$K \times 10^5$ C.G.S. ...	1·767	1·349	1·253

These results are indicated by circles in the graphs given below in which the curves already published in 1916 (loc. cit.) are extended so as to include the saturated solutions studied in the present research.

V. *Definite Concentration.*

The value of the Diffusivity obtained above is that for the solution in the diffusion tube, which varies in concentration from saturation to zero. This coefficient the author has called the "Mean Diffusivity" and has shown that the coefficient of Diffusion, K_n, for a solution of a definite concentration can be calculated theoretically from the value of the Mean Diffusivity by a formula (No. 6) given on p. 56 in

the Paper referred to. In applying this calculation to the present case it is advisable as before to include the term containing n^2, and we obtain

$$K+\{1-\tfrac{1}{2}n\delta+\tfrac{1}{3}n^2\delta\,(\delta-b)\}/\{1-n\delta+n^2\delta\,(\delta-b)\}\,ndK/dn$$
$$=c/\{1-n\delta+n^2\delta\,(\delta-b)\}\,dl/dn$$
$$=K_n.$$

The following results are obtained by this method for the coefficient of Diffusion of saturated solutions of the salts mentioned at the temperature indicated :—

Salt.	Temperature.	$K_n\times10^5$.
KCl	17·3°C.	2·107 C.G.S.
NaCl	17·1	1·522
KNO_3	18·1	1·171

These results are also included in the graphs and are indicated by crosses. The curves associated with them show the value of the coefficient of diffusion at other definite concentrations calculated in the same way. It will be observed that both the Mean Diffusivity and the Diffusion Coefficient for the saturated solutions employed agree to within 1 per cent. with the corresponding values obtained by extrapolation from the results previously found for less concentrated solutions, and an inspection of the curves shows that the range of concentration over which the author has studied diffusion by this method has been approximately doubled by the work included in this Paper.

The research has been carried out at Birkbeck College, and I should like to record my continued indebtedness to Dr. A. Griffiths, in whose department the work was done, for his kind interest and assistance.

DISCUSSION.

Dr. A. Griffiths dealt with the fundamental expression for diffusion. He explained the theoretical foundation of the diffusion-equation used by Mr. Clack and himself, and stated that this equation was an extension of Fick's Equation, or that Fick's Equation could be considered to be a limiting case of their equation. He said that Fick's Equation implicitly assumes that the liquid is at rest during a diffusion operation, whereas in fact the changes in concentration in all experiments known to himself inevitably produce movement of the liquid. An extension of Fick's hypothesis is, therefore, inevitable. He explained that the equation used by Mr. Clack and himself assumed that in addition to the diffusion as expressed by Fick, there is a movement of the diffusing salt equal in value to that of the water-component of the salt solution. He stated that the quantity defined by δ in earlier Papers by Mr. Clack and himself should have been defined as the ratio of the *mass* of water entering the diffusing cell to the

mass of salt leaving the cell. The modification of the definition makes no change whatever when a linear relation may be assumed between density and concentration; and, in any case, makes a change smaller than errors of experiment. The change becomes smaller with the concentration, and is zero at infinite dilution.

The coefficients of diffusion for the weakest solutions of potassium and of sodium chloride were each 16 per cent. less than the theoretical values given by the Nernst equation for infinitely dilute solutions. The value of the coefficient for potassium nitrate was 11 per cent. less than the theoretical. These differences could not be considered very remarkable if it were remembered that Perrin's value (based on work with liquids) for the electronic charge was 11 per cent. less than the generally accepted value. The fundamental hypotheses for liquids probably require a close examination. One of his students had interpreted Mr. Clack's results by the equation $K=K_1r+K_2(1-r)$, where K is the coefficient obtained by Mr. Clack, K_1 is the coefficient of diffusion of the completely ionised salt, K_2 is the coefficient of diffusion of the non-ionised salt, and r is the ionisation-ratio. In the case of potassium nitrate this equation represents the results obtained by Mr. Clack wonderfully well over the whole range of concentration, and it represents the results well in the case of potassium chloride and of sodium chloride when the concentrations are not great.

Capt. C. W. Hume did not think the concentration at the top of the diffusion tube would be zero. The effect of this error would be equivalent to a constant increase in the length of the tube.

Mr. F. E. Smith asked if experiments could not be made by measuring differences in electrical conductivity. Such methods, it appeared to him, would enable results to be obtained much more rapidly.

Mr. Clack, in reply, said that earlier in his researches he had investigated the end-correction suggested by Mr. Hume and concluded that it was negligible. As regards the conductivity method, this had been tried, but he was at present experimenting with an optical method with which it should be possible to obtain results for all concentrations in one experiment.

XXIV. *Experiments on Thermal Transpiration Currents.* By Gilbert D. West, *D.Sc.* (*Lond.*).

Received January 4, 1921.

ABSTRACT.

This Paper is an attempt to demonstrate the actual existence of thermal transpiration currents. Theoretical considerations are first introduced to show that if a radial temperature gradient be maintained over a disc, so that the centre is the hottest part, thermal transpiration currents sweep radially inwards over the surface of the disc, and discharge themselves more or less radially outwards in the upper regions.

To detect these currents a narrow strip of foil is used which is placed perpendicular to the disc, and to one side of the hot region. When at a considerable perpendicullar distance from the disc, and when the gas pressure is sufficiently low to eliminate convection currents, the deflections of the strip of foil are always away from the hot region. When, however, the strip is placed very close to the disc, its deflections over a certain range of gas pressure are towards the hot region. These facts are explained by the tendency of the thermal transpiration currents to drag the strip with them.

The Paper emphasises one of the essential differences between thermal transpiration currents and convection currents—namely, that while the latter clearly depend on gravitation, the former do not. Further emphasis is laid on the differences between the conditions of molar and molecular equilibrium.

Introduction.

In a previous Paper to the Physical Society* the author found it possible to explain the peculiar movements of heated strips of foil in rarefied gases, by means of thermal transpiration currents. Moreover, it was found possible to give a satisfactory explanation of the Crookes radiometer on a similar basis.† The actual existence of these currents was thus strongly favoured, and, in the present work, some of the phenomena to which they should give rise are discussed and conclusions drawn. These conclusions are then submitted to experimental tests.

General Considerations.

It is first necessary to realise that the properties of a gas vary according as we consider it from the molar or molecular point of view. To make matters clear, take the case of a vessel filled with gas and separated into two compartments, *A* and *B*, by a partition in which there is a small hole. If the diameter of the hole—although small compared with the

* Proc. Phys. Soc., Vol. XXXII., p. 166, 1920.
† Proc. Phys. Soc., Vol. XXXII., p. 222, 1920.

dimensions of the vessel—be large compared with the mean free path of the molecules, the gas can be considered to behave as a continuous viscous fluid, and if, for instance, a difference of temperature be established between the two compartments, a flow of gas will take place from the hot to the cold compartment. In the equilibrium state, small disturbances due to convection currents may take place at the orifice, but, but for these, the pressures in the two vessels will be equal.

Let us now consider the case when the hole is small compared with the mean free path of the molecules. It is clear that we cannot now regard the gas as a continuous fluid. On the contrary, everything must be considered from the molecular standpoint, and equilibrium will therefore only be reached when the numbers of molecules passing per second in opposite directions through the hole are the same. The consequent relation between the pressures in the two compartments is easily deduced if we divide the molecules up into the six conventional Joulean sets, and imagine that there are $1/6NV$ impacts per sec. per square cm. of wall, where N represents the number of molecules per cubic centimetre, and V the root mean square velocity. Clearly, equilibrium can only be reached when

$$\tfrac{1}{6}N_A V_A = \tfrac{1}{6}N_B V_B,$$

where the suffixes distinguish the molecules in the two compartments. Hence, if m be the mass of a molecule, the ratio of the pressures P_A and P_B will be given by

$$P_A/P_B = \tfrac{1}{3}N_A m V_A^2 / \tfrac{1}{3}N_B m V_B^2 = V_A/V_B = \sqrt{T_A}/\sqrt{T_B},$$

where T_A and T_B represent the respective absolute temperatures.

The distribution of pressure is thus seen to be different according as the size of the hole is large or small compared with the mean free path of the molecules. It will be convenient to use the terms "molar" and "molecular" equilibrium to distinguish these two extreme cases. Intermediately, when the conditions are neither molecular nor molar, complex phenomena occur, and no very general cases of equilibrium under these conditions appear to have been dealt with satisfactorily. Sutherland,* however, has considered the case of a tube along which a uniform temperature gradient is maintained, and which connects hot and cold vessels con-

* Phil. Mag., 42, p. 373, and p. 476, 1896.

taining gas. It is necessary even here to simplify the problem, and to restrict calculations to pressures at which convection currents are unimportant. When this reservation is made, it is found that there can be considered to exist two superposed systems of gas currents. The first, termed a thermal transpiration current, is from the cold to the hot vessel, is uniform over the whole section, and is similar to the flow that would take place in the simple case just discussed, were we to start with the hot and cold compartments A and B at the same pressure. It is clear that such a current exhibits the extraordinary power of flowing from a region of low pressure to a region of high pressure. This, however, is not surprising if we regard it as an attempt to establish molecular equilibrium.

Superposed on this uniform thermal transpiration current, and discharging an equal amount of gas per second, is another current in the reverse direction, which follows the laws of viscous flow in a tube, and which thus flows from regions of high pressure to regions of low pressure, and which rises from a minimum value at the walls to a maximum at the axis. This current, of course, can be regarded as an attempt to establish molar equilibrium.

The result of the superposition of the two flows is to give a current from the cold to the hot vessel along the walls of the tube, together with a current from the hot to the cold vessel along the axis, whilst between the two there will be a surface of zero velocity.

At very high pressures, when molar conditions are approached, or at very low pressures, when molecular conditions are approached, the gas currents will be very feeble. On the other hand, at some intermediate pressure, when the conditions are neither molar nor molecular, the currents will be at a maximum. Starting from the lowest pressures, therefore, the velocity of the gas currents will rise in value with inrceasing pressure, reach a maximum when the mean free path is of the order of the diameter of the tube, and finally, at higher gas pressures, decline asymptotically to zero.*

It is to be noted that, whether the equilibrium is molar or molecular, or intermediate between the two, equilibrium is conditioned by the equality of the number of molecules

* On the basis of a previous Paper (Proc. Phys. Soc., Vol. XXXI., p. 279, 1919) there is no difficulty in obtaining an expression for the distribution velocity of the gas over the cross-section of the tube. For any given point the velocity is proportional to dp/dx, as obtained from equation (2), p. 285. Note, however that 65·1 should be replaced by 71·9.

passing in opposite directions. The only difference is the way in which this may be accomplished. Under molar conditions it may be accomplished by oppositely directed currents, but, under molecular conditions, the gas has lost all resemblance to a continuous fluid, and such currents are no longer possible.

Let us now turn our attention to the variation of the excess pressure in vessel A with gas pressure. It has already been shown that at the lowest gas pressures

$$P_A/P_B=\sqrt{T_A}/\sqrt{T_B}.$$

If T_A and T_B are close we may drop suffixes and write

$$\text{Excess pressure in vessel } A=\delta p=\frac{\delta T}{2T}\cdot p.$$

It is thus seen that when molecular equilibrium is approached the excess pressure in A is proportional to the gas pressure. On the other hand, it can be shown* that when the equilibrium

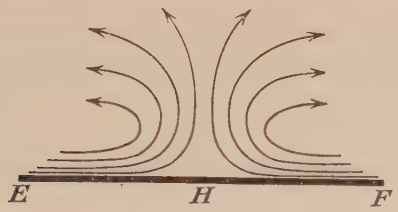


FIG. 1.—DISTRIBUTION OF GAS CURRENTS AROUND A DISC EF WITH CENTRAL HOT REGION H.

is nearly molar, the excess pressure in A is inversely proportional to the gas pressure. In fact, the function representing variation of excess pressure in A with gas pressure, rises, reaches a maximum, and again declines.

Thus both the gas currents and the excess pressures in the hot compartment follow the same type of variation. In a tube, moreover, both the maxima occur at the same gas pressure, but for more complicated cases this is not necessarily the case.

Let us now leave the case of the tube and try to extend our considerations to a case that lends itself to easy experimental verification, namely, that of a disc over whose surface a radial temperature gradient is maintained, so that the centre is the

* Proc. Phys. Soc., Vol. XXXI., p. 279, 1919.

hottest part. We should here expect thermal transpiration currents to flow radially inwards over the surface, and to discharge themselves more or less radially outwards, as in Fig. 1.

Let us further consider the effect of the introduction of a thin metal strip, *AB* (Fig. 2), perpendicular to the disc *EF*,

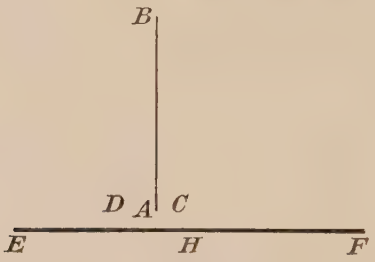


Fig. 2.—*EF* Represents a Disc with Central Hot Region *H*, and *AB* a Strip of Thin Metal Leaf.

and to one side of the hot region *H*. The mean temperature of region *C* will clearly be above that of region *D*, and this region of higher temperature must, according to principles previously enunciated, be associated with a higher pressure. If we start from the lowest gas pressures, this excess pressure will at first rise with increasing gas pressure, then reach a

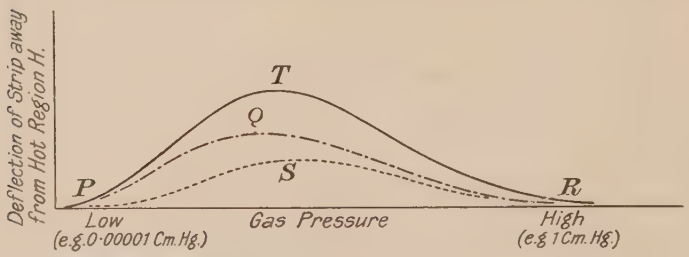


Fig. 3.—Indicating Deflections of Strip at Various Gas Pressures—Strip Distant from Disc.

maximum, and finally decline asymptotically to zero. In fact, it might quite well be represented graphically by the curves *PQR* in Figs. 3 or 4. Moreover, such excess pressures will be capable of producing deflections of the strip away from the hot region *H*.

The gas currents that sweep over the surface of the disc,

however, are also capable of producing deflections of the strip, but the curve which will represent deflections of the strip due to this agency will depend on the perpendicular distance of the strip from the disc. If the strip is far away from the disc, the strip will merely have its deflections augmented by the outflowing currents, and we might well represent such augmentations by *PSR* (Fig. 3). If, on the other hand, the strip is close to the disc, the end *A* will tend to be driven towards the hot region by the inflowing gas currents, and we might now imagine such deflections represented by *PSR* (Fig. 4). It will be noted that we here suppose the maxima in the curves *PSR* (Figs. 3 and 4) to occur at higher pressures than in the curves *PQR*. This is admitted to be little more

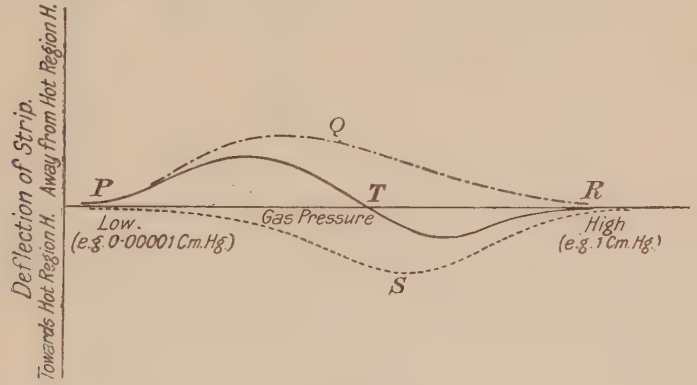


FIG. 4.—INDICATING DEFLECTIONS OF STRIP AT VARIOUS GAS PRESSURES —STRIP CLOSE TO DISC.

than a probable assumption, but if this assumption be made, the resultant deflections will be represented by *PTR* (Figs. 3 and 4), and it will be noted that, in one case, the deflections are always in the same direction, whereas, in the other, they change in direction with gas pressure.

If now the thermal transpiration currents actually exist, and if they function in the way indicated, experiment should bear out these curves. In what follows, therefore, a set of experiments is described, and the results obtained are considered.

Apparatus.

The apparatus consisted of a cylindrical brass tube about 9 cm. in diameter, at the ends of which were cemented two

glass plates, V and W (Fig. 5). To V was fixed a thin vertical disc EF of aluminium foil about 0·001 cm. thick, and, close to the centre of the disc, was fixed, in a horizontal plane, a small strip of aluminium leaf AB about 0·71 cm. long and 0·2 cm. broad. It was found that, owing to its slight curvature of section, AB would remain horizontal although EF was vertical. Radiation from a "Pointolite" lamp P

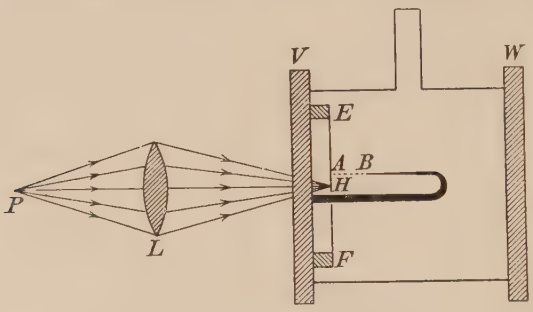


Fig. 5.—Diagram of Apparatus.

was concentrated at H by means of a lens system L, and the box could be exhausted by means of a Gaede pump, and the pressure measured by McLeod gauges. Observations of the small deflections of the strip were made by a microscope, through a glass window in the side of the box.

Results.

Results obtained with the strip AB at 0·29 cm. from the screen, and with the hot region in succession above and below

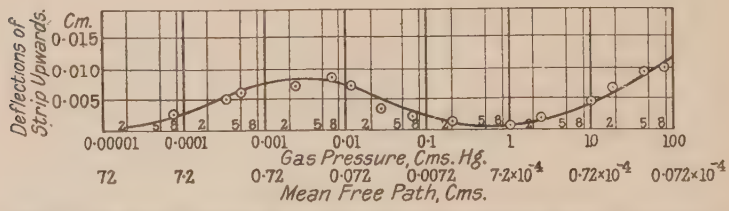


Fig. 6.—Showing Variation of Strip Deflections with Gas Pressure. (Hot region below strip. Distance of end of strip from screen 0·29 cm.)

the strip, are shown in Figs. 6 and 7. It will be seen that, below 1 cm. of mercury, the curve rises and falls in a way similar to PTR (Fig. 3), and that in each case the deflection is away from the hot region. Deflections upwards, however,

are greater than deflections downwards, and this may be attributed to the slight concavity of the upper surface of the strip, and its consequent resistance to downward deflections.

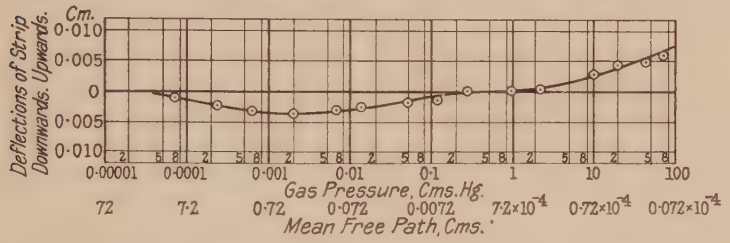


FIG. 7.—SHOWING VARIATION OF STRIP DEFLECTIONS WITH GAS PRESSURE. (Hot region above strip. Distance of end of strip from screen 0·29 cm.)

In Figs. 8 and 9 deflections are shown with the strip much closer to the screen. At the lowest pressures, the deflections

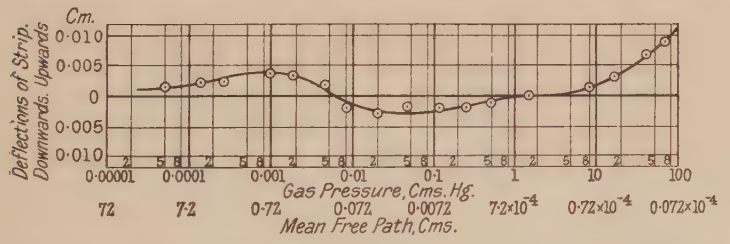


FIG. 8.—SHOWING VARIATION OF STRIP DEFLECTIONS WITH GAS PRESSURE. (Hot region below strip. Distance of end of strip from screen 0·09 cm.)

are in the same directions as before, but, with increase of pressure, the deflections are in each case reversed. The

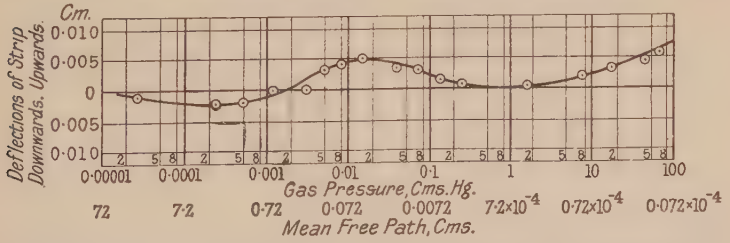


FIG. 9.—SHOWING VARIATION OF STRIP DEFLECTIONS WITH GAS PRESSURE. (Hot region above strip. Distance of end of strip from screen 0·09 cm.)

general shapes of the experimental curves, from the lowest pressures to 1 cm. mercury, are, in fact, strikingly similar to

that of the theoretical curve *PTR* of Fig. 4. We cannot, therefore, help feeling some confidence in the assumptions made in deducing it. In particular, a good case for the actual existence of thermal transpiration currents can be made out.

Above 1 cm. of mercury deflections also occur, and these increase with increasing pressure, but do not change in direction when the position of the hot region is changed. There is but little doubt that these deflections are due almost entirely to convection currents streaming vertically upwards over the disc.

Convection currents are, in fact, characteristic of molar equilibrium, and, with reduction of gas pressure, they quickly tend to disappear, whilst their place is taken by thermal transpiration currents. These thermal transpiration currents, although resembling convection currents in that they are associated with temperature inequalities, are differentiated from such convection currents by their independence of gravitation. They are essentially transition phenomena that are important when the mean free path of the molecules is too long for the conditions to be considered molar, yet too short for the conditions to be considered molecular.

In conclusion, the author has pleasure in thanking Professor Lees for the facilities afforded for prosecuting this research.

DISCUSSION.

Dr. D. OWEN said the author appeared to be working down to pressures of a ten-thousandth of a centimetre of mercury. How were these pressures measured ?

Dr. WEST.—With a McLeod gauge.

Dr. OWEN then asked if the author considered these to be reliable at such low pressures.

Dr. WEST replied that he always used two gauges and they gave similar results. The bulbs were from 200 c.c. to 300 c.c. capacity. He thought if care was taken to use perfectly dry air the gauges were quite reliable.

THE SIXTH GUTHRIE LECTURE.

Some Recent Applications of Interference Methods.

Delivered by PROF. A. A. MICHELSON, *University of Chicago.*

MARCH 11, 1921.

FOR some years it has been my hope to have an opportunity of putting into execution three investigations involving application of interference of light waves: first, a repetition of the measurement of earth tides; second, a redetermination of the velocity of light; third, an attempt to measure the diameter of a star.

Through the kindness of Dr. George E. Hale such an opportunity was presented in the form of an invitation to attempt all three problems at Pasadena, California, with the aid of the splendid equipment of the Observatory at Mount Wilson and of the laboratory facilities at Pasadena.

EARTH TIDES.

The first of these problems had already been brought to a satisfactory conclusion at Yerkes Observatory, with the result that the earth tides were very nearly three-tenths of the calculated value of the water tides—leading to a value for the rigidity of the earth about the same as that of steel. One object in repeating this investigation at Pasadena was the possibility of finding difference between a station well in the interior of the continent and another quite near the coast.

The method consists in the measurement of the difference in level of water in a steel pipe 6 in. in diameter, 100 ft. long, buried 6 ft. below the surface and hermetically sealed. It was found that under these conditions the effects due to changes in temperature, wind or other atmospheric disturbances were practically eliminated.

In a preliminary trial, lasting over three months, it was found that the results of measurements by the microscope (pointing on a needle point under the water surface, and on its totally reflected image) were so concordant and so little affected by earth tremors that it was decided to employ the interferometer. In the usual form, the instrument consists essentially of a plane surface upon which the incident light falls at an angle of 45 deg., part passing through and part reflected, thus separating the original pencil into two at right angles. These are reflected back on their paths, the

first by a mirror in air, the second by a mirror very near and below the water surface. The two paths being the same with the exception of the thin film of water, the variation in this thickness causes a displacement of the resulting interference bands; and knowing the wave length of the incident light (the violet radiation from a mercury lamp) and the index of refraction of the water, the corresponding change of level can readily be calculated.

In the present determination a simpler arrangement is

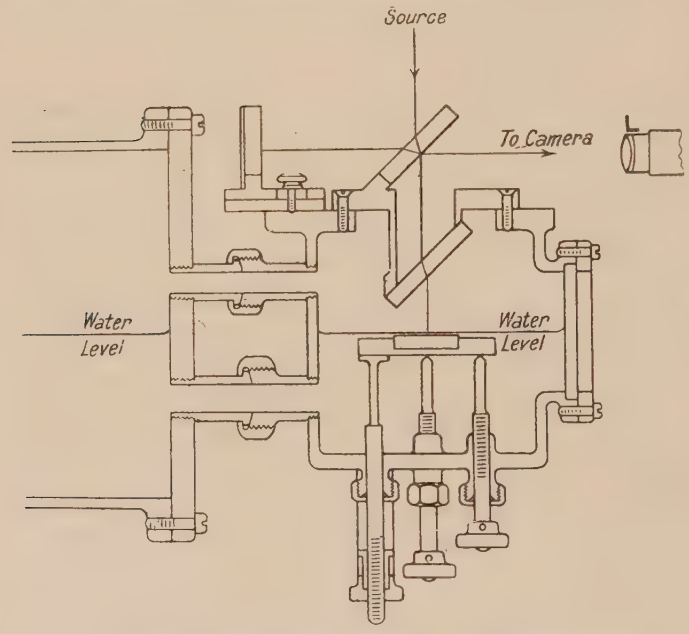


FIG. 1.—EARTH TIDES. ARRANGEMENT OF INTERFEROMETER.

adopted, namely, a plane glass surface just below the water surface, between which two surfaces the interference of the reflected light occurs.

With a tube 100 ft. long this gives a difference in level due to the tidal forces of sun and moon, of an amplitude of the order of 10 to 20 fringes, the position of which can be readily measured to within one-tenth of a fringe, thus giving an average error less than 1 per cent.

A continuous photographic record of the interference bands at both ends of the east-west line, and also the north-

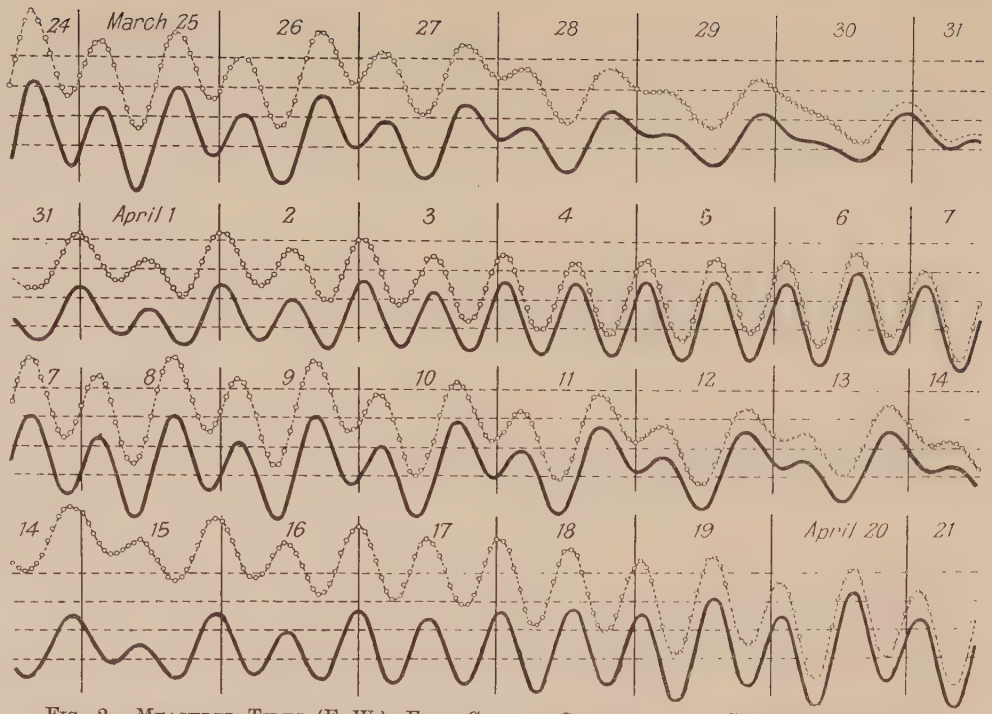


FIG. 2.—MEASURED TIDES (E.-W.), FULL CURVE; COMPARED WITH CALCULATED VALUES DIMINISHED IN PROPORTION OF 7 TO 10.

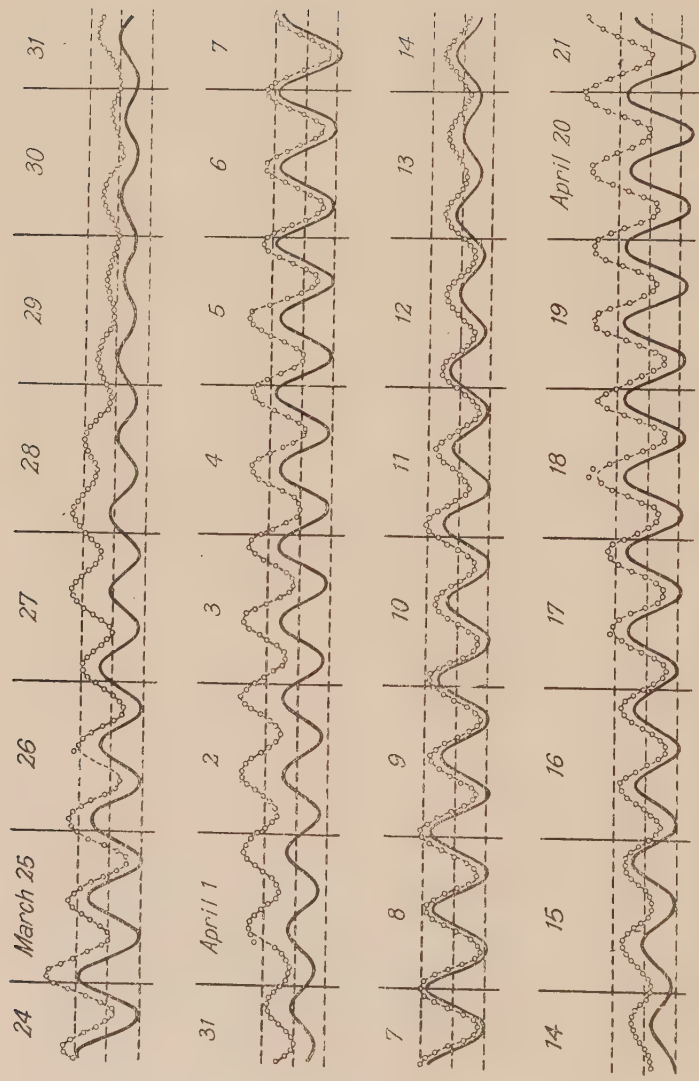


Fig. 3.—Measured Tides (N.S.), Full Curve; Compared with Calculated Values Diminished in Proportion of 7 to 10.

south line gave results agreeing almost exactly with the calculated values when these are diminished in the ratio of

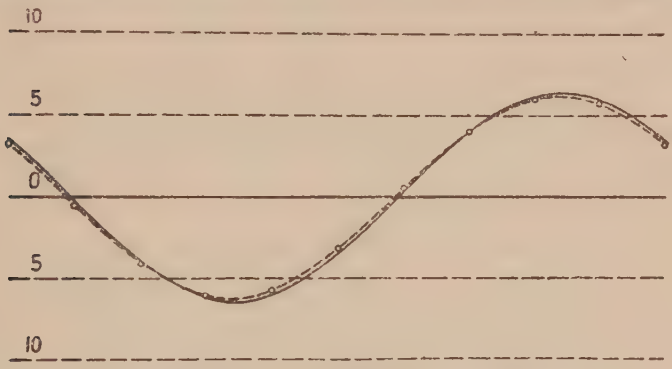


FIG. 4.—N.-S. SEMI-DIURNAL LUNAR TIDE, FOR ENTIRE YEAR.

Dotted curve from observed values
$y = 6{\cdot}173 \sin(\theta + 148°24')$

Full curve from 0·7 times calculated values
$y = 6{\cdot}401 \sin(\theta + 147°32')$
$R = 0{\cdot}7\ a/b = 0{\cdot}675.\quad \Delta\phi = 52$

7 to 10. There is, however, a phase retardation amounting to about 4 deg., which may be interpreted as a result of viscosity, or as due to the effect of the ocean tides; and it is

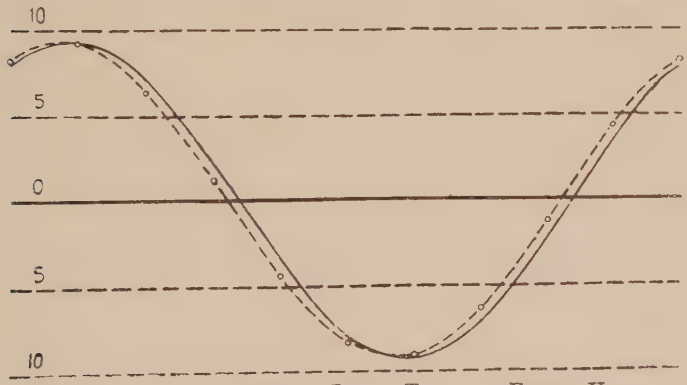


FIG. 5.—E.-W. SEMI-DIURNAL LUNAR TIDE, FOR ENTIRE YEAR.

Dotted curve from observed values
$y = 9{\cdot}380 \sin(\phi + 63°56')$

Full curve from 0·7 of calculated values
$y = 9{\cdot}485 \sin(\theta + 58°10')$
$R = 0{\cdot}7\ c/d = 0{\cdot}692.\quad \Delta\varphi = 5°46'$

probable that in the present determination (near the coast) this retardation may be found to have a different value.

NOTE.—It may be mentioned that in this investigation a continuous record of the interference bands was obtained for a period of an entire year—interrupted, however, during three hours by an earthquake found to have occurred at a distance of 2,000 miles. It appears thus that the installation may serve as a seismograph; and it may even be possible to observe a slow secular change in the apparent level due to an inclination of the rock strata, the ultimate rupture of which causes the earthquake.

THE VELOCITY OF LIGHT.

In view of the increasing importance of the velocity of light as a fundamental constant of Nature, and in view of its relation to the ratio between the electromagnetic and the electrostatic units, it does not seem superfluous to repeat the measurement—especially as it seems possible, by a modification of the method of the revolving mirror, to obtain an order of accuracy at least 10 times as great as that hitherto reached.

Thus, while by the usual method the accuracy of the result depends upon three factors namely, the distance, the speed of the revolving mirror, and the angular deflection of the returned ray, the last measurement is in the method proposed eliminated, or preferably transferred to the fixed angles between the faces of the revolving mirror.

It is in this last process that the application of interference comes into play.

If the revolving mirror be given the form of an octagon, the angles being all equal, and if the mirror revolve at, say, 1,000 turns per second, it would require $\frac{1}{8000}$ of a second for one face to replace the preceding one; and in this time light travels 375 km.; so that the distance between stations may be under 20 km. Or, with 40 km., the speed need be only 500 turns per second.

In order to secure equality between the angles of the octagon, these are compared by interference tests with a standard angle of 135 deg. When the difference is the same for all, they must, of course, be equal to 135 deg. In the mirror actually constructed in this manner, the average error has been found to be of the order of one part in a million.*

* It may be worth pointing out that this may furnish a method for dividing a circle to the same order of accuracy.

The experiments have only been carried far enough, at a distance of four miles, to indicate a strong probability of success at a much greater distance, and it is hoped that the investigation may be continued during the coming summer.

The Diameter of Stars.

The application of interference to astronomical measurements was indicated in an article published some 30 years ago ("Phil. Mag.," July, 1890): a trial of the method applied to the measurement of the diameter of Jupiter's satellites gave results of an order of accuracy hardly to be attained by the use of the large telescopes of to-day. It was then pointed out that the method might give important results in the measurement of double stars, and even in estimating the diameter of some of the nearer stars if these should be considerably larger than our sun.

Briefly, the method (at least, in the earlier and simpler form) consists in employing only two small portions of the objective, at opposite ends of a diameter, instead of the whole objective.

In both cases it is, of course, an interference pattern which is observed; but whereas with the complete objective this is a bright "disc" surrounded by a number of coloured rings, in the former case it will consist of a number of coloured bands symmetrically arranged about the central achromatic band. This if the source is a point.

But if the source is a double star, then, as is well known, these can be "resolved," using the full objective, when by Rayleigh's formula, $\alpha = 1{\cdot}22\lambda/d$, where α is the angular separation, λ the mean wave-length and d the diameter of the objective. But the actual "resolution" will depend a good deal on the observer, and in still greater degree on the atmospheric disturbances, which, indeed, are usually so strong as to obliterate all traces of the diffraction fringes.

In the proposed method, if d now represents the distance between the apertures, the formula becomes $\alpha = 0{\cdot}5\lambda/d$, so that, apart from other considerations, the accuracy is more than doubled. But here it is not a more or less vague appearance of "resolution" which is observed, but an actual vanishing of the fringes (supposing the components of the double star to be of equal brightness), and this vanishing can be observed with remarkable accuracy, so that, in fact, the increase in accuracy is at least tenfold.

The same considerations can be shown to apply, with

modification, to the measurement of the diameter of a star; in this case the vanishing of the interference fringes occurs when $\alpha = 1{\cdot}22\lambda/d$, α now representing the angular diameter.

Before making an actual attempt to obtain measurements, it was clearly desirable to ascertain whether atmospheric disturbances, which make good "seeing" so exceptional, would not have a still more drastic influence on the interference fringes by the new method. Accordingly, observations were

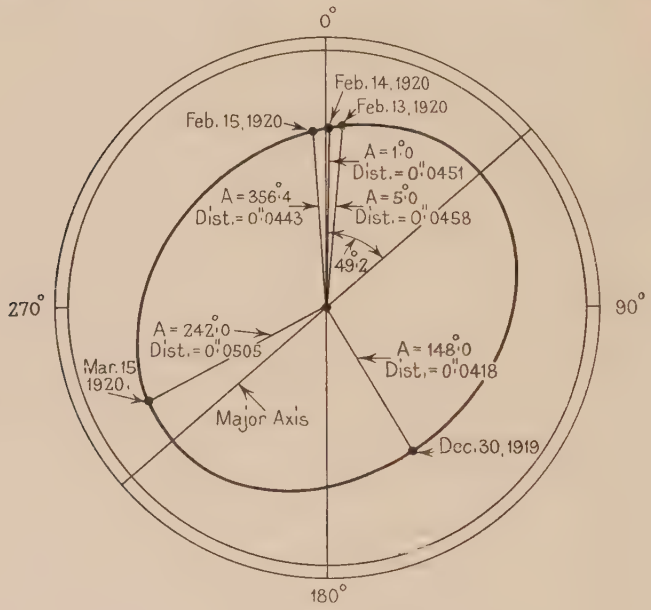


FIG. 6.—VISUAL ORBIT OF CAPELLA.

made using the 40 in. refractor of the Yerkes Observatory, with the result that fringes were obtained even when the seeing was but 2 on a scale of 5.

A still more severe test was made at the Mount Wilson Observatory, first with the 60 in. and then with the 100 in. reflector.* In both cases the fringes were remarkably clear even when the seeing was 2 on a scale of 10.

The explanation of this very satisfactory but rather un-

* The screen with apertures in the case of the 100 in. was applied only a short distance from the focus, instead of in front of the objective, which made measurements much more convenient.

expected result is probably as follows: At any very brief instant the fringes should be distinct. At any subsequent brief instant they should be equally clear, but in a different position; but if the motion is not too rapid for the eye to follow, they remain distinct at all times, whereas in the use of the entire objective there is an integrated disturbance which may obliterate the diffraction fringes entirely.

Shortly after these tests, Mr. Anderson made a series of observations on Capella, which was known to be a double from spectroscopic evidence, and succeeded in measuring both distance and position angle so accurately that the differences between the observations and the computed orbit differed by quantities of the order of one ten thousandth of a second, the whole distance being of the order of 0·″04.

The Double Star Capella. Observed and Computed Orbits.

Date of Observation.	Distance.	Position Angle.
	″	°
1.—December 30·6, 1919	0·0418	(153·9)
2.—February 13·6, 1920	0·0458	5·0
3.—February 14·6, 1920	0·0451	1·0
4.—February 15·6, 1920	0·0443	356·4
5.—March 15·6, 1920	0·0505	242·0
6.—April 23·6, 1920	(0·0439)	107·0

$t-T$.	a.	i.	r.	Pos. Angle.
d	″	° ° ′	″	°
1.—39·7 ...	1 and 5 0·05249	180—39 31	0·04180	153·9
2.—84·7 ...	2 and 5 0·05250	—39 36	0·04583	4·6
3.—85·7 ...	3 and 5 0·05248	—39 22	0·04506	1·0
4.—86·7 ...	4 and 5 0·05249	—39 31	0·04430	357·3
5.—11·7 ...	———	—39 30	0·05050	242·4
6.—50·7 ...	———	———	0·04391	107·2
	Means 0·05249	180—39 30		

It was then proposed, under the direction of Mr. Pease, to build an interferometer, with a base of 20 ft. in the form of a structural iron beam attached to the 100 in. tube, and provided with two plane mirrors at 45 deg., one at each end of the beam, which reflect the light from the star radially toward the axis of the tube. Two similar mirrors near the axis thence reflect the two pencils down the tube to the 100 in. mirror, whence they proceed to the focus, producing the interference fringes which are observed under rather high magnification (about 2,000 diameters).

In order to secure equality in path, a double wedge of glass

is inserted in the path of one of the pencils, compensated by a plane parallel plate in the other path, and the wedges moved by a slow motion screw until, with the aid of a small direct-vision

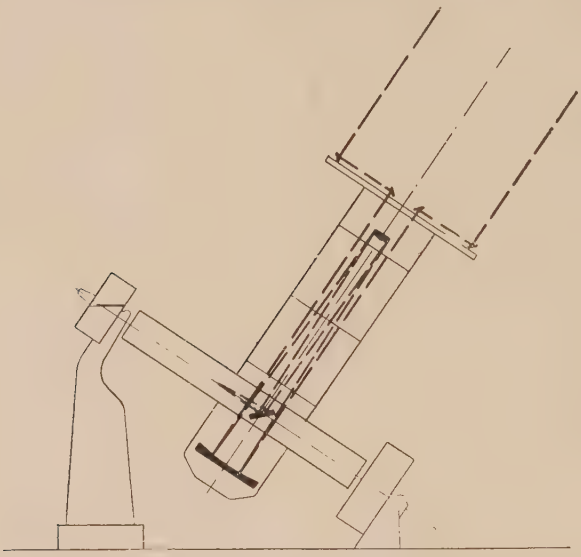


Fig. 7.—Illustrating Path of the Two Interfering Pencils of Light from a Star.

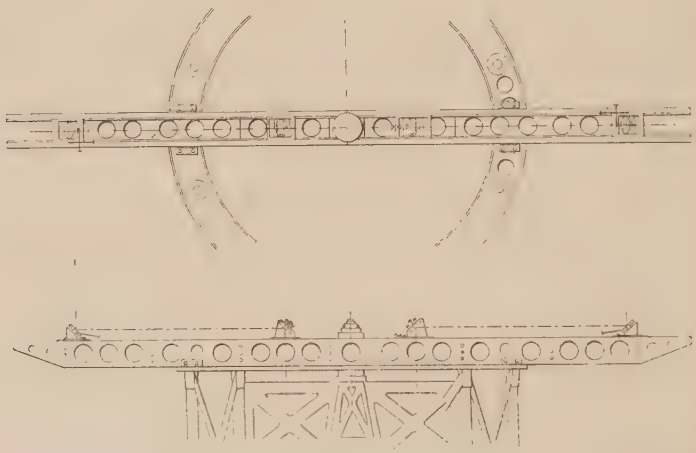


Fig. 8.—Plan and Elevation of Interferometer of 20 ft. Base Attached to Tube of 100 in. Reflector.

prism at the eyepiece, the resulting spectrum is traversed by dark bands, when the prism is removed and the wedges still further adjusted until interference fringes appear in white light.

These were, in fact, observed in August, 1920. In December the actual vanishing of the interference fringes was observed by Pease when pointing at α-Orionis, the distance between the outer mirrors being 10 ft., while, as a check, fringes were still visible on adjacent stars. It follows from the known parallax $\pi=0{\cdot}016$, that the diameter of this star is about 300 times as great as that of the sun, which confirms in a remarkable degree the estimates given by Eddington and Russell.

The construction of an interferometer with a base of 50 ft. to 100 ft. is contemplated, which may make it possible to measure the diameter of Sirius.

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